We consider a mechanical system with n degree-of-freedom whose generalized coordinates are q_1, q_2, ..., q_n. The Lagrange equations describing the motion of the system are

$$\frac{d}{dt}(\frac{\partial \mathcal{L}}{\partial \dot{q}_r}) - \frac{\partial \mathcal{L}}{\partial q_r} = \tau_r, \quad r = 1, 2, ..., n, \tag{1}$$

where $\mathcal{L} = T - P$, T and P are the kinetic and potential energy respectively, and τ_r is the generalized force. The kinetic energy can be expressed as

$$T = \frac{1}{2}\sum_{i,j=1}^{n} a_{ij}\dot{q}_i\dot{q}_j \tag{2}$$

where a_{ij} is a function of the generalized coordinates.

In the following, we denote $\mathbf{q} = [q_1 \ q_2 \ ...q_n]^T$, then Lagrange's equation (1) can be manipulated to derive

$$D(\mathbf{q})\ddot{\mathbf{q}} + F(\dot{\mathbf{q}}, \mathbf{q})\dot{\mathbf{q}} + G(\mathbf{q}) = \tau \tag{3}$$

where the $n \times n$ matrix $D(\mathbf{q})$ is positive definite and symmetric, and is related to the inertial properties of system, the vector function $F(\dot{\mathbf{q}}, \mathbf{q})\dot{\mathbf{q}}$ is in general a nonlinear function of its arguments and $\tau = [\tau_1, \ \tau_2, ...\tau_n]^T$.

In the following development we only consider the systems with the following two simplifying properties.

Property 1: A suitable definition of $F(\mathbf{q}, \dot{\mathbf{q}})$ makes the matrix $(\dot{D} - 2F)$ skew-symmetric. In particular, this is true if the elements of $F(\mathbf{q}, \dot{\mathbf{q}})$ are defined as

$$F_{ij} = \frac{1}{2}[\dot{\mathbf{q}}^T \frac{\partial D_{ij}}{\partial \mathbf{q}} + \sum_{k=1}^{n}(\frac{\partial D_{ik}}{\partial q_j} - \frac{\partial D_{jk}}{\partial q_i})\dot{q}_k] \tag{4}$$

Property 2: There exists a m-vector α with components depending on mechanical parameters (masses, moments of inertia, etc.), such that

$$D(\mathbf{q})\ddot{\mathbf{q}} + F(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} + G(\mathbf{q}) = \Phi(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}})\alpha \tag{5}$$

where Φ is a $n \times m$ matrix of known functions of $\mathbf{q}$, $\dot{\mathbf{q}}$, $\ddot{\mathbf{q}}$; and α is the m-vector of inertia parameters.

The control objective can be specified as: given desired $\mathbf{q}_d$, $\dot{\mathbf{q}}_d$, and $\ddot{\mathbf{q}}_d$, which are assumed to be bounded, determine a control law for τ such that $\mathbf{q}$ asymptotically converge to $\mathbf{q}_d$, satisfying the following conditions:

- All the parameters in the manipulator system (3) are unknown

- Use only joint position measurements
- Bounded torque inputs are used (an explicit bounds on the torques should be delivered)

It is necessary to use a two-link manipulator with the parameters given to verify your control algorithm.