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RESEARCH PAPER

Output Feedback Adaptive Fractional-Order Super-Twisting Sliding Mode Control of Robotic Manipulator

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Abstract

In this study, trajectory tracking of robotic manipulators with uncertainties and external disturbances is obtained by proposing model-free adaptive fractional super-twisting sliding mode control (AOFSTSM). The proposed AOFSTSM method is composed of an adaptive super-twisting sliding mode control integrated with fractional-order (FO) control. An adaptive tuning control is utilized to evaluate the uncertain unknown dynamics of the system without relying on the prior knowledge of the upper bounds. Moreover, FO control and super-twisting sliding mode control are used to achieve the fast finite-time convergence, chatter-free control inputs, better tracking performance and robustness. An output feedback (OF) is proposed and the state estimation is obtained by robust exact differentiator. Furthermore, the stability of the overall system is investigated and derived from the Lyapunov stability criterion. Finally, to validate the effectiveness and robustness of the developed control method, comparative simulations of state-feedback and OF of proposed method with fractional-order nonsingular fast terminal sliding mode control are realized to demonstrate the performance of AOFSTSM.

Keywords Adaptive control · Output feedback control · Super-twisting sliding mode control · Robotic manipulator · Fractional-order control

1 Introduction

In recent years, considerable efforts are taken for controlling the robot manipulators which require high control performance in precision and accuracy of the joint position tracking. The robot manipulator is a time-varying nonlinear system and subject to the problems such as model uncertainties, parameter variations and external disturbances. If these problems are not properly controlled, then, it may deteriorate system performance and stability of the closed-loop system.

It is worth noting that several control schemes have been proposed to deal with uncertainties and parameter variations for instance: adaptive fuzzy sliding mode control (Beyhan et al. 2011), H_∞ adaptive control (Wang et al. 2014; Ahmed et al. 2019a), neural network control (Richa et al. 2016), fractional-order PID control (Ayad et al. 2019) etc. Based on H_∞ technique, a robust adaptive method was developed using known regression matrix of 2-degree of freedom (DOF) robotic manipulator (Wang et al. 2014). Therefore, it is challenging and sometimes impossible to formulate the regression matrix for more than 2-DOF robotic manipulator and other nonlinear systems. Thus, it is the demanding task for the research scientist to propose a controller to achieve precise joint positioning of unknown dynamic of the robot manipulator without using the regression matrix.

In the literature, several model-independent control schemes were proposed such as recursive model-free control (Wang et al. 2011), time delay control (TDC) (Wang et al. 2018; Ahmed et al. 2018a), iPD/ iPI/ iPID (Michel and Cédric 2013) and so on. In Wang et al. (2011), the recursive control method was designed for the linearized model of the inverted pendulum. On the other hand, nonlinear model of robotic manipulator was considered and controlled by TDC and iPD/ iPI/ iPID. Although these schemes were simple and easy to implement, however, recursive model-free control did not consider uncertainties,

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while iPID control performances may be deteriorated by numerical instabilities in algebraic loop. And the performance of TDC may be degraded due to inserted constant delay (Wang et al. 2020a).

On the other front, adaptive control can be preferred and used to estimate the unknown uncertain system dynamics. Adaptive control is a notable nonlinear control approach which updates the control gain according to the current situation of the closed-loop system. Moreover, this control scheme is broadly used to control linear and nonlinear systems (Tao 2014). The adaptive control has also been integrated with traditional and advanced control schemes such as PID (Sangpet et al. 2018), sliding mode control (Hsu et al. 2003), optimal control (Aldair et al. 2019), fuzzy control (Doudou and Khaber 2019), and neural network control (Wei et al. 2016). Sliding mode control is strongly robust against uncertainties, while adaptive control can deal with unknown system dynamics. Thus, adaptive control with SMC approach is used to simultaneously obtain the advantages of both control schemes such as robustness against uncertainties, estimate the unknown dynamics and update the control gain online (Yen et al. 2019; Ahmed et al. 2019c; Mobayen et al. 2019).

Sliding mode control (SMC) is an important branch of the variable structure control (VSC), VSC is renowned as insensitive to system uncertainties and parameter variations. SMC was widely used for the satellite systems (Han et al. 2016), robotic manipulator (Wang et al. 2020b), oscillators (Solis et al. 2017), power electronics applications (Utkin 2013), etc. However, traditional SMC had some drawbacks such as large steady-state error, slow response, singularity problem and chattering phenomena in the control inputs. To handle these problems, terminal SMC (TSMC) was developed which employs nonlinear surface and guarantees the robust stability, and fast convergence (Van et al. 2017; Venkataraman and Gulati 1993; Zhang et al. 2019). However, TSMC still has chattering problem which can affect the closed-loop system performance.

Various control methods were proposed to eliminate the chattering problem such as by replacing the *sign* function to saturation or sigmoid function (Van et al. 2017; Ma and Sun 2018; Aghababa et al. 2011; Aghababa 2014). High-order SMC (HOSMC) was also utilized to reduce the chattering problem efficiently (Wang et al. 2020b; Shtessel and Tournes 2009). Moreover, another high-order SMC technique is introduced in Feng et al. (2014), where third-order TSMC was developed to obtain chatter-free control input. Meanwhile, to improve the HOSMC, super-twisting algorithm has been proposed to eliminate the chattering problem and guarantees the high control performance simultaneously (Wang et al. 2019).

Fractional-order (FO) calculus is an old mathematical analysis that dates to the 17th century. FO calculus is the generalization of classical differentiation and integration to non-integer order (Tavazoei et al. 2008). In recent decades, the application of FO in the field of science and engineering gained the attention of the researchers (Podlubny 1998). It provides more flexibility to improve the control performance, and various FO based controllers such as FO-PID controllers and FO adaptive controllers have been proposed (Ayad et al. 2019; Wang et al. 2016). Moreover, FO with TSMC methods was proposed to enhance the system performance in terms of fast convergence and high trajectory tracking with small steady-state error (Hosseini and Mohammad 2017; Nojavanzadeh and Mohammadali 2016; Ahmed et al. 2019b). In some cases, the states such as velocity and acceleration are unavailable or cannot be measured. Thus, to deal with this situation, well-suited observers or differentiators can be used to estimate the system states.

To date, adaptive schemes use the regression matrix and known system model for the controller design (Wang et al. 2014; Nojavanzadeh and Mohammadali 2016; Ahmed et al. 2018b). In Wang et al. (2014), robotic manipulator was controlled using H_∞-Adaptive control scheme, where complex calculation of regression matrix was required. Moreover, adaptive SMC approach was designed for the known dynamics of robotic manipulator (Nojavanzadeh and Mohammadali 2016). To the best of our knowledge, how to design an adaptive controller in the absence of such information (regression matrix and system parameters) has not yet been addressed in the literature. Thus, there is strong motivation to propose a model-free adaptive control design method which can guarantee the system performance without knowledge of the system dynamics. The main contributions of this work are given as follows:

1. The novel output feedback model-free adaptive control method with super-twisting SMC (AOFSTSM) is presented without the calculation of the regression matrix.
2. Model-free control is obtained by adaptive control design and STSM provides the robustness against uncertainties and disturbances.
3. It provides simple and easier implementation without prior knowledge of system dynamics.
4. A proof of stability is investigated by Lyapunov stability synthesis.

The following paper is arranged as: Sect. 2 introduces the important preliminaries. The control objective is given in Sect. 3. Section 4 presents the designing of the proposed AOFSTSM method and the stability analysis of the closed-loop system. The comparative analyses of numerical simulations demonstrated in Sect. 5 to illustrate the effectiveness of AOFSTSM. In the end, this paper is summarized in Sect. 6.

2 Preliminaries

In this section, preliminaries regarding FO calculus and finite-time convergence are given.

Definition 1 The Riemann–Liouville (RL) fractional derivative and integral of β_{th}-order of function *f(t)* with respect to t and the terminal value a are given by Li et al. (2017)

$$ {}_a\mathcal{D}_t^{\beta} f(t) = \frac{\mathrm{d}^{\beta} f(t)}{\mathrm{d}t^{\beta}} = \frac{1}{\Gamma(r-\beta)} \frac{\mathrm{d}^r}{\mathrm{d}t^r} \int_a^t \frac{f(\tau)}{(t-\tau)^{\beta-r+1}} \mathrm{d}\tau \tag{1} $$

$$ {}_a\mathcal{D}_t^{-\beta} f(t) = {}_a\mathcal{I}_t^{\beta} f(t) = \frac{1}{\Gamma(\beta)} \int_a^t \frac{f(\tau)}{(t-\tau)^{1-\beta}} \mathrm{d}\tau \tag{2} $$

where $r-1<\beta<r$, while $\mathcal{D}^{\beta}$ and $\mathcal{I}^{\beta}$ denote the fractional derivative and integral, respectively. And $\Gamma(\cdot)$ is Euler's Gamma function given by

$$ \Gamma(\beta) = \int_0^{\infty} e^{-t} t^{\beta-1} \mathrm{d}t \tag{3} $$

Lemma 1 *The n_{th} order derivative $(\mathrm{d}^n/\mathrm{d}t^n)$ of the fractional derivative operator ${}_a\mathcal{D}_t^{\beta} f(t)$ can be transformed as Nojavanzadeh and Mohammadali (2016)*

$$ \frac{\mathrm{d}^n}{\mathrm{d}t^n}\left({}_a\mathcal{D}_t^{\beta} f(t)\right) = {}_a\mathcal{D}_t^{\beta}\left(\frac{\mathrm{d}^n f(t)}{\mathrm{d}t^n}\right) = {}_a\mathcal{D}_t^{\beta+n} f(t) \tag{4} $$

Lemma 2 *For Lyapunov function $\mathcal{V}(t)$ with initial value $\mathcal{V}(t_0)$, finite-time stability can be computed as Aghababa et al. (2011)*

$$ \dot{\mathcal{V}}(t) \le -n\mathcal{V}^p(t), \quad \forall t \ge t_0, \ \ \mathcal{V}(t_0) \ge 0 \tag{5} $$

where $n>0$ and $0<p<1$. *Therefore, the finite-time t_f can be formulated as*

$$ t_f \le \frac{1}{n(1-p)} \mathcal{V}^{1-p}(t_0) \tag{6} $$

3 Control Objective

For the applicability of proposed AOFSTSM method, the appropriate form of the uncertain dynamics of n-DOF robot manipulator in the presence of uncertainties and external disturbances is expressed in this section. The n-DOF robot manipulator dynamics are given by

$$ M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) + d = \tau \tag{7} $$

where q, $\dot{q}$ and $\ddot{q} \in \Re^n$ represent the angular position, velocity and acceleration, respectively. $M(q) \in \Re^{n\times n}$ denotes the positive definite inertia matrix, $0<\underline{\lambda}(M(q)) \le \|M\| \le \bar{\lambda}(M(q))$, where $\underline{\lambda}$ and $\bar{\lambda}$ denote the minimum and the maximum eigenvalues, respectively. $C(q,\dot{q}) \in \Re^{n\times n}$ expresses the coriolis and centripetal forces, $G(q) \in \Re^n$ symbolizes the gravitational matrix, $\tau \in \Re^n$ and $d \in \Re^n$ denote the joint control torque and the unknown external disturbances, respectively.

Therefore, one can rewrite (7) in the following form as

$$ \ddot{q} + \mathcal{F}(q,\dot{q},\ddot{q}) = N^{-1}\tau \tag{8} $$

where
$\mathcal{F}(q,\dot{q},\ddot{q}) = N^{-1}(M(q)\ddot{q} - N\ddot{q} + C(q,\dot{q})\dot{q} + G(q) + d)$,
N is the positive diagonal matrix.

Remark 1 Since only the joint position can be measured using encoders, therefore, velocity and acceleration can be computed by robust exact differentiation (RED) approach given by Levant (2003)

$$ \begin{aligned} \dot{z}_0 &= -\beta_1 |z_0 - q|^{2/3}\mathrm{sign}(z_0 - s) + z_1 \\ \dot{z}_1 &= -\beta_2 |z_1 - \dot{z}_0|^{1/2}\mathrm{sign}(z_1 - \dot{z}_0) + z_2 \\ \dot{z}_2 &= -\beta_3 \mathrm{sign}(z_2 - \dot{z}_1) \end{aligned} \tag{9} $$

where $z_0 = q, z_1 = \dot{q}, z_2 = \ddot{q}$ and $\beta_1, \beta_2, \beta_3$ are positive constants.

For the tracking error of the system, (8) can be written as

$$ \ddot{e} = N^{-1}\tau - \mathcal{F}(q,\dot{q},\ddot{q}) - \ddot{q}_d \tag{10} $$

where $e = z_0 - q_d$, $\dot{e} = z_1 - \dot{q}_d$ and $\ddot{e} = z_2 - \ddot{q}_d$ represent the error and its derivative terms, and q_d, $\dot{q}_d$ and $\ddot{q}_d$ are the desired inputs. Therefore, the objective is to design a suitable robust fractional super-twisting terminal sliding mode controller for an unknown system that ensures z_0 converges to q_d, which implies error trajectories converge to sliding manifold $s(t) = 0$ in the finite-time t_f.

Assumption 1 The unknown dynamics are bounded as follows

$$ |\mathcal{F}(q,\dot{q},\ddot{q})| \le \Xi(z_0,z_1)^{\mathrm{T}}\varsigma \tag{11} $$

where $\Xi(z_0,z_1) = \left[1,\ |z_0|,\ |z_1|^2\right]^{\mathrm{T}}$, and $\varsigma = [\varsigma_1, \varsigma_2, \varsigma_2]^{\mathrm{T}}$ is finite positive constant vector.

4 Controller Development

In this section, the proposed method is formulated, which includes the following steps: initially, super-twisting sliding mode scheme with FO control is developed. Later, to obtain model-free control, an adaptive fractional-order super-twisting sliding mode control is proposed named AOFSTSM. Then, stability analysis of the overall system is validated by the application of Lyapunov theorem.

4.1 Fractional Super-twisting Sliding Mode Control Design

To define FO sliding surface, firstly, the STSM surface is designed as follows

$$s = \dot{e} + \bar{k}_1 \mathcal{D}^{\beta-1} \mathrm{sgn}(e)^{\upsilon} + \bar{k}_2 e \tag{12}$$

where $s = (s_1, s_2, \cdots s_n)^{\mathrm{T}}$, $\mathrm{sgn}(e)^{\upsilon} = |e|^{\upsilon}\mathrm{sign}(e)$ with $0<\upsilon<1$, $\mathcal{D}^{\beta}$ is the fractional derivative, β is fractional-order ranges between $0<\beta<1$ and $\bar{k}_1 \in \Re^{n\times n}$ and $\bar{k}_2 \in \Re^{n\times n}$ are positive diagonal matrix.

By taking the derivative of s, one obtains

$$\dot{s} = \ddot{e} + \bar{k}_1 \mathcal{D}^{\beta} \mathrm{sgn}(e)^{\upsilon} + \bar{k}_2 \dot{e} \tag{13}$$

Substitution of (10) into (13), one gets

$$\dot{s} = N^{-1}\tau - \mathcal{F}(q, \dot{q}, \ddot{q}) - \ddot{q}_d + \bar{k}_1 \mathcal{D}^{\beta} \mathrm{sgn}(e)^{\upsilon} + \bar{k}_2 \dot{e} \tag{14}$$

The above proposed surface obtains highly robust control accuracy, precise position tracking and fast convergence speed can be efficiently endured. To reduce chattering and obtain good performance in the reaching phase, the super-twisting scheme is used with (12) as follows

$$\begin{aligned} \dot{s} &= -k_1 \mathrm{sgn}(s)^{1/2} + \eta \\ \dot{\eta} &= -k_2 \mathrm{sgn}(s) \end{aligned} \tag{15}$$

After proposing the sliding manifold, the following controller based on fractional super-twisting SMC is designed as

$$\tau = N[\tau_{\mathrm{nom}} + \tau_{\mathrm{stw}}] \tag{16}$$

where τ_{nom} is the control input employed to deal with the known components, therefore, the corresponding equation of τ_{nom} is formulated as follows

$$\tau_{\mathrm{nom}} = N[\ddot{q}_d - \bar{k}_1 \mathcal{D}^{\beta} \mathrm{sgn}(e)^{\upsilon} - \bar{k}_2 \dot{e} + \Xi(z_0, z_1)^{\mathrm{T}} \varsigma] \tag{17}$$

while τ_{stw} is based on super-twisting sliding surface and obtains chatter-free control inputs. Thus, τ_{stw} is given as follows

$$\tau_{\mathrm{stw}} = -k_1 \mathrm{sgn}(s)^{1/2} - k_2 \int_0^t \mathrm{sgn}(s)\mathrm{d}t \tag{18}$$

where $k_1 = diag(k_{11}, k_{12} \cdots k_{1n})$ and $k_2 = diag(k_{21}, k_{22} \cdots k_{2n})$ are positive matrix. Substitution of (16) into (14), one has

$$\begin{aligned} \dot{s} =& \Xi(z_0, z_1)^{\mathrm{T}} \varsigma - \mathcal{F}(q, \dot{q}, \ddot{q}) - k_1 \mathrm{sgn}(s)^{1/2} \\ & - k_2 \int_0^t \mathrm{sgn}(s)\mathrm{d}t \end{aligned} \tag{19}$$

The closed-loop model of the proposed method is shown in Fig. 1. Now the stability analysis of sliding mode dynamics will be presented in the following theorem.

Theorem 1 *If the sliding mode dynamics (14) and the control law (16) are designed, then it ensures the stability of the nonlinear system converging to origin along* $s = 0$.

Proof The following Lyapunov functional candidate is given as

$$V(\xi) = \left(2k_2 + \frac{k_1^2}{2}\right)\xi_1^2 - k_1 \xi_1 \xi_2 + \xi_2^2 \tag{20}$$

where

$$\xi = [\,\xi_1 \quad \xi_2\,]^{\mathrm{T}} = \left[\,\mathrm{sgn}(s)^{1/2} \quad \eta\,\right]^{\mathrm{T}}$$

. The above equation (20) can be rewritten as

$$V(\xi) = \xi^{\mathrm{T}} K \xi \tag{21}$$

with $K = \frac{1}{2}\begin{bmatrix} 4k_2 + k_1^2 & -k_1 \\ -k_1 & 2 \end{bmatrix}$. the matrix K is the positive definite such that

$$\lambda_{\min}(K)\|\xi\|^2 \le V(\xi) \le \lambda_{\max}(K)\|\xi\|^2 \tag{22}$$

where $\lambda_{\min}(K)$ and $\lambda_{\max}(K)$ are minimum and maximum eigenvalues of K. $\|\xi\|$ is Euclidean norm which can be expressed as $\|\xi\|^2 = |\xi| + \xi_2^2$.

By taking derivative of ξ, one gets

$$\dot{\xi} = [\,\dot{\xi}_1 \quad \dot{\xi}_2\,]^{\mathrm{T}} = \left[\,\frac{\dot{s}}{2|s|^{1/2}} \quad \dot{\eta}\,\right]^{\mathrm{T}} \tag{23}$$

Substituting (15) into (23), one can obtain

$$\dot{\xi} = \frac{1}{|s|^{1/2}} A\xi,\ A = \begin{bmatrix} -\frac{k_1}{2} & \frac{1}{2} \\ -k_2 & 0 \end{bmatrix}.$$

The time derivative of $V(t)$ can be computed as follows

$$\dot{V}(\xi) = \dot{\xi}^{\mathrm{T}} K \xi + \xi^{\mathrm{T}} K \dot{\xi} \tag{24}$$

Substitution of $\dot{\xi}$ into (24), one can derive

$$\dot{V}(\xi) \le -\frac{1}{|s|^{1/2}} \xi^{\mathrm{T}} Q \xi \tag{25}$$

where

$$Q = A^{\mathrm{T}} K + KA = \frac{k_1}{2}\begin{bmatrix} (2k_2 + k_1^2) & -k_1 \\ -k_1 & 1 \end{bmatrix} > 0$$

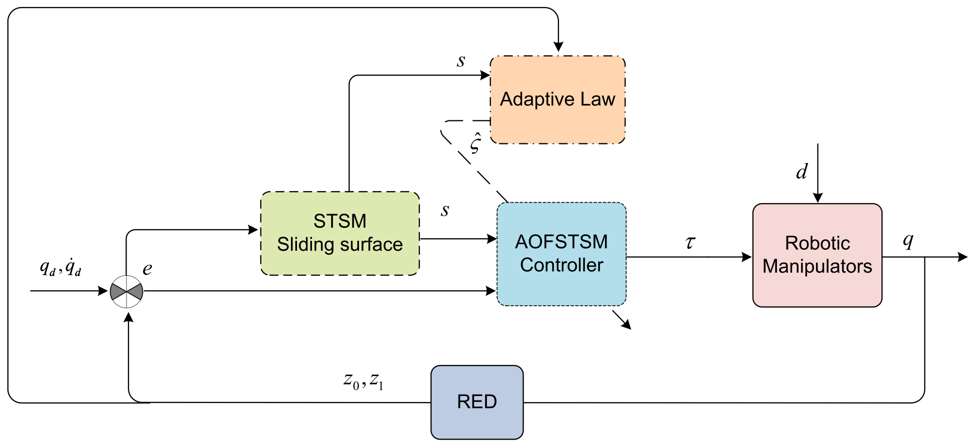

Fig. 1 Complete proposed AOFSTSM control model

$$\dot{V}(\xi) \leq -\frac{1}{|s|^{1/2}}\lambda_{\min}(Q)\|\xi\|^2 \tag{26}$$

Combining (22) and (26) and the fact that $|\xi_1| = |s|^{1/2} \leq \|\xi\|_2 \leq \frac{V^{1/2}(\xi)}{\lambda_{\min}^{1/2}(K)}$, one can get

$$\dot{V}(\xi) \leq -\frac{\lambda_{\min}^{1/2}(K)\lambda_{\min}(Q)}{\lambda_{\max}(K)} V^{1/2}(\xi) \tag{27}$$

$$\dot{V}(t) \leq -\psi V^{1/2}(\xi) \tag{28}$$

where $\psi = \frac{\lambda_{\min}^{1/2}(K)\lambda_{\min}(Q)}{\lambda_{\max}(K)}$.

Hence, the proof shows that the stability is ensured and the nonlinear system will reach the origin along $s = 0$ in finite time. To calculate the finite-time, according to Lemma 1, corresponding finite-time can be calculated as follows

$$t_f \leq \frac{2V^{\frac{1}{2}}(\xi)}{\psi} \tag{29}$$

□

Remark 2 From Eqs. (16) and (29), convergence time and control input are influenced by the constant parameters k_1 and k_2. Hence, a tradeoff between the convergence speed and control performance is required to select a suitable value of gains.

4.2 Adaptive Fractional Super-Twisting Sliding Mode Control Design

The control scheme in Sect. 4.1 is model-based where the system is considered to be known and bounded. In case of unknown system dynamics, it is challenging to obtain system information and design control law. Thus, adaptive control with OFSTSM (AOFSTSM) is proposed to deal with uncertain unknown system dynamics under external disturbances.

Thus, the following model-free AOFSTSM approach is proposed to control for the unknown system dynamics as

$$\bar{\tau} = \tau_{adp} + \tau_{\mathrm{stw}} \tag{30}$$

$$\tau_{adp} = N[\ddot{q}_d - \bar{k}_1 \mathcal{D}^{\beta} \mathrm{sgn}(e)^{\upsilon} - \bar{k}_2 \dot{e} + \Xi(z_0, z_1)^{\mathrm{T}} \hat{\varsigma}] \tag{31}$$

where τ_{stw} is same as defined in (16), and τ_{adp} is the control input which includes adaptive parameters to estimate the unknown system dynamics. Therefore, the adaptive parameter $\hat{\varsigma}$ is the estimations of unknown constants ς defined in Assumption 1.

Substitution of (30) into (14), yields

$$\dot{s} = -k_1 \mathrm{sgn}(s)^{1/2} + \eta - \mathcal{F}(q, \dot{q}, \ddot{q}) + \Xi(z_0, z_1)^{\mathrm{T}} \hat{\varsigma} \tag{32}$$

Using Assumption 1, the above equation can be rewritten as

$$\dot{s} = -k_1 \mathrm{sgn}(s)^{1/2} + \eta + \Xi(z_0, z_1)^{\mathrm{T}} \bar{\varsigma} \tag{33}$$

where $\bar{\varsigma} = \hat{\varsigma} - \varsigma$.

For updating the parameters in (32), the following adaptive law is designed as

$$\dot{\hat{\varsigma}} = \begin{cases} 0 & if \;\; |s| \leq \zeta \\ -\gamma \Xi(z_0, z_1) B^{\mathrm{T}} K \xi & if \;\; |s| > \zeta \end{cases} \tag{34}$$

where $\zeta > 0$ is dead-zone size to avoid parameter drifting problem and $\gamma > 0$ is tuning constant gain.

Theorem 2 *For the nonlinear system dynamics of robot manipulators* (7), *the fractional super-twisting terminal sliding surface is introduced in* (12), *and the finite-time*

Table 1 2-DOF robotic manipulator parameters

	mass-m (kg)	Inertia-I (kg m^2)	length-l (m)
$link-1$	0.1	0.1	1
$link-2$	0.1	5	0.85

convergence is ensured under the robust control law (30) *and the adaptive law* (34).

Proof The appropriate Lyapunov functional candidate is selected as follows

$$V(\xi) = \xi^{\mathrm{T}} K \xi + \frac{1}{2\gamma} \bar{\varsigma}^{\mathrm{T}} \bar{\varsigma} \tag{35}$$

The derivative of ξ under unknown dynamics is given by

$$\dot{\xi} = \frac{1}{|s|^{1/2}} A\xi + \frac{1}{2} B \Xi(z_0, z_1)^{\mathrm{T}} \bar{\varsigma} \tag{36}$$

with

$$A = \begin{bmatrix} -\frac{k_1}{2} & \frac{1}{2} \\ -k_2 & 0 \end{bmatrix}, B = \begin{bmatrix} 1/|s|^{1/2} \\ 0 \end{bmatrix}.$$

Now the derivative of $V(\xi)$ can be obtained as follows

$$\dot{V}(\xi) = \dot{\xi}^{\mathrm{T}} K \xi + \xi^{\mathrm{T}} K \dot{\xi} + \frac{1}{\gamma} \bar{\varsigma}^{\mathrm{T}} \dot{\bar{\varsigma}} \tag{37}$$

Substitution of (36) into (37), one can get

$$\begin{aligned} \dot{V}(\xi) \leq & \frac{1}{|s|^{1/2}} \xi^{\mathrm{T}} (A^{\mathrm{T}} K + KA) \xi + \bar{\varsigma}^{\mathrm{T}} \Xi(z_0, z_1) B^{\mathrm{T}} K \xi \\ & + \frac{1}{\gamma} \bar{\varsigma}^{\mathrm{T}} \dot{\bar{\varsigma}} \end{aligned} \tag{38}$$

By simplifying above equation, we have

$$\dot{V}(\xi) \leq -\frac{1}{|s|^{1/2}} \xi^{\mathrm{T}} Q \xi + \bar{\varsigma}^{\mathrm{T}} \left(\Xi(z_0, z_1) B^{\mathrm{T}} K \xi + \frac{1}{\gamma} \dot{\bar{\varsigma}} \right) \tag{39}$$

By substituting adaptive law (34), one obtains

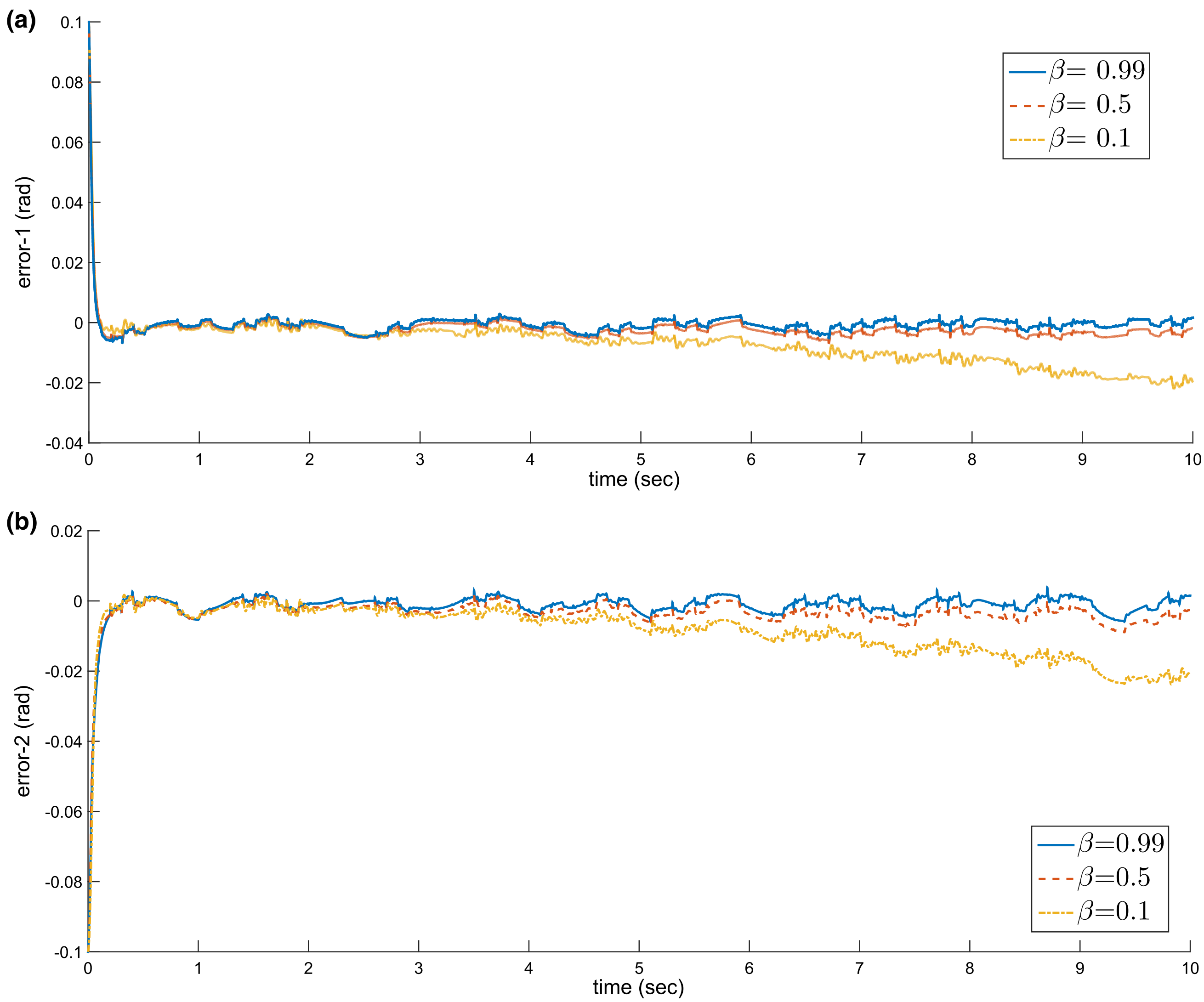

Fig. 2 Tracking error under different value of β

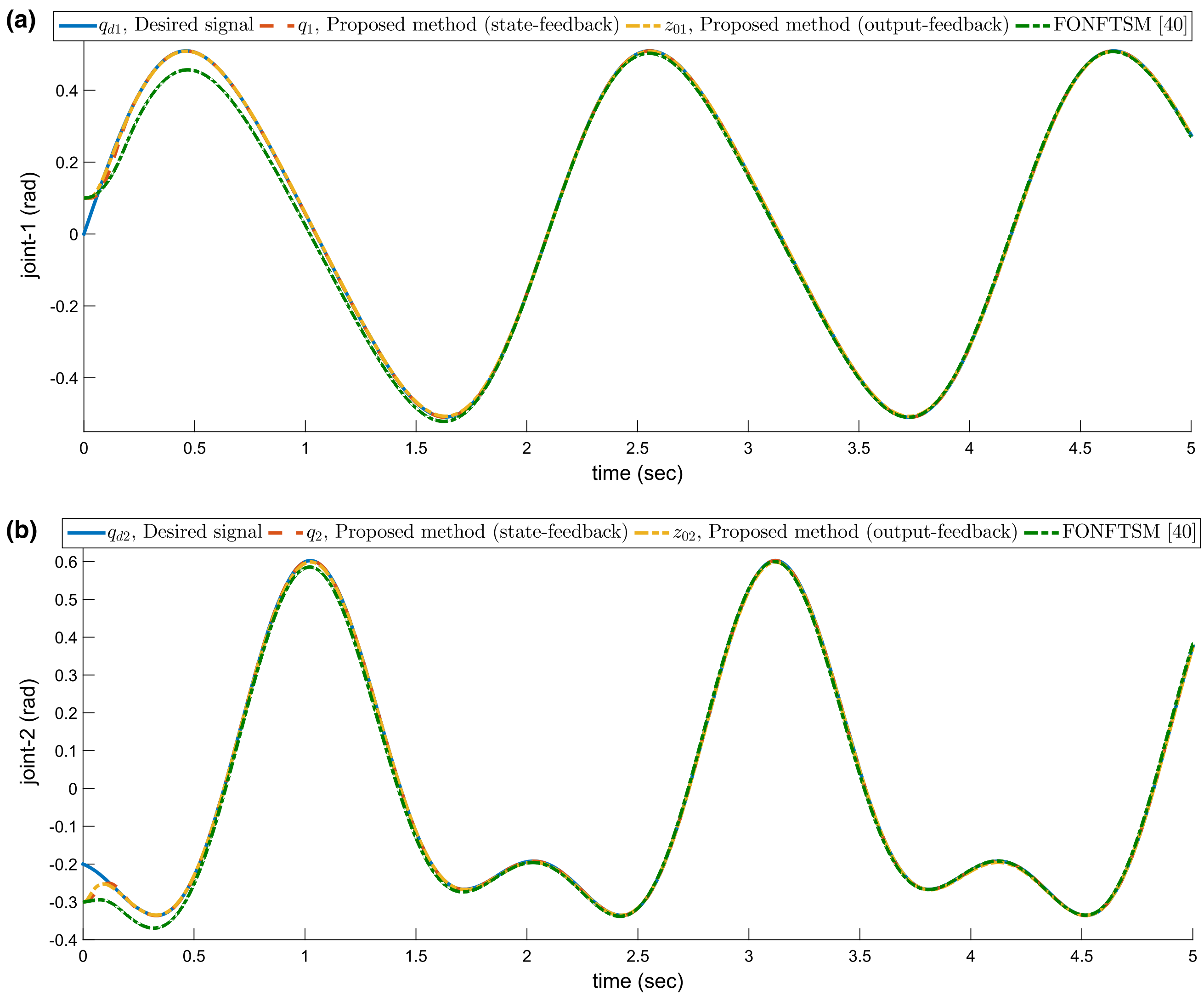

Fig. 3 Position tracking without measurement noise

$$\dot{V}(\xi) \leq -\frac{1}{|s|^{1/2}}\xi^{\mathrm{T}}Q\xi \tag{40}$$

Since $Q > 0$, Thus, the system will converge to the sliding manifold robustly. Hence, the stability proof of the overall system is completed. Moreover, the formulation of finite-time will be similar as given in Theorem 1. □

Remark 3 The parameters of the proposed AOFSTSM scheme are selected according to the specified range which is defined as $\bar{k}_1 > 0, \bar{k}_2 > 0, k_1 > 0, k_2 > 0, N > 0, 0 < \beta < 1, 0 < \upsilon < 1$ and $K > 0$. Thus, if the parameters are not selected within the specified range then the stability of the closed-loop system cannot be achieved. Hence, to obtain the desired trajectory tracking and closed-loop system stability simultaneously, the suitable value can be selected accordingly.

5 Numerical Simulations

In this section, an uncertain 2-DOF robotic manipulator under external disturbances is controlled using the proposed AOFSTSM method. To validate the effectiveness of the proposed method, comparative analyses are demonstrated between state-feedback (SF) and output-feedback (OF) of the proposed schemes and FONFTSM (Ahmed et al. 2018b) under measurement noise.

5.1 Dynamics of 2-DOF Robotic Manipulators

The model of 2-DOF robotic manipulators and its dynamic equation under external disturbances are described as follows Wang et al. (2016)

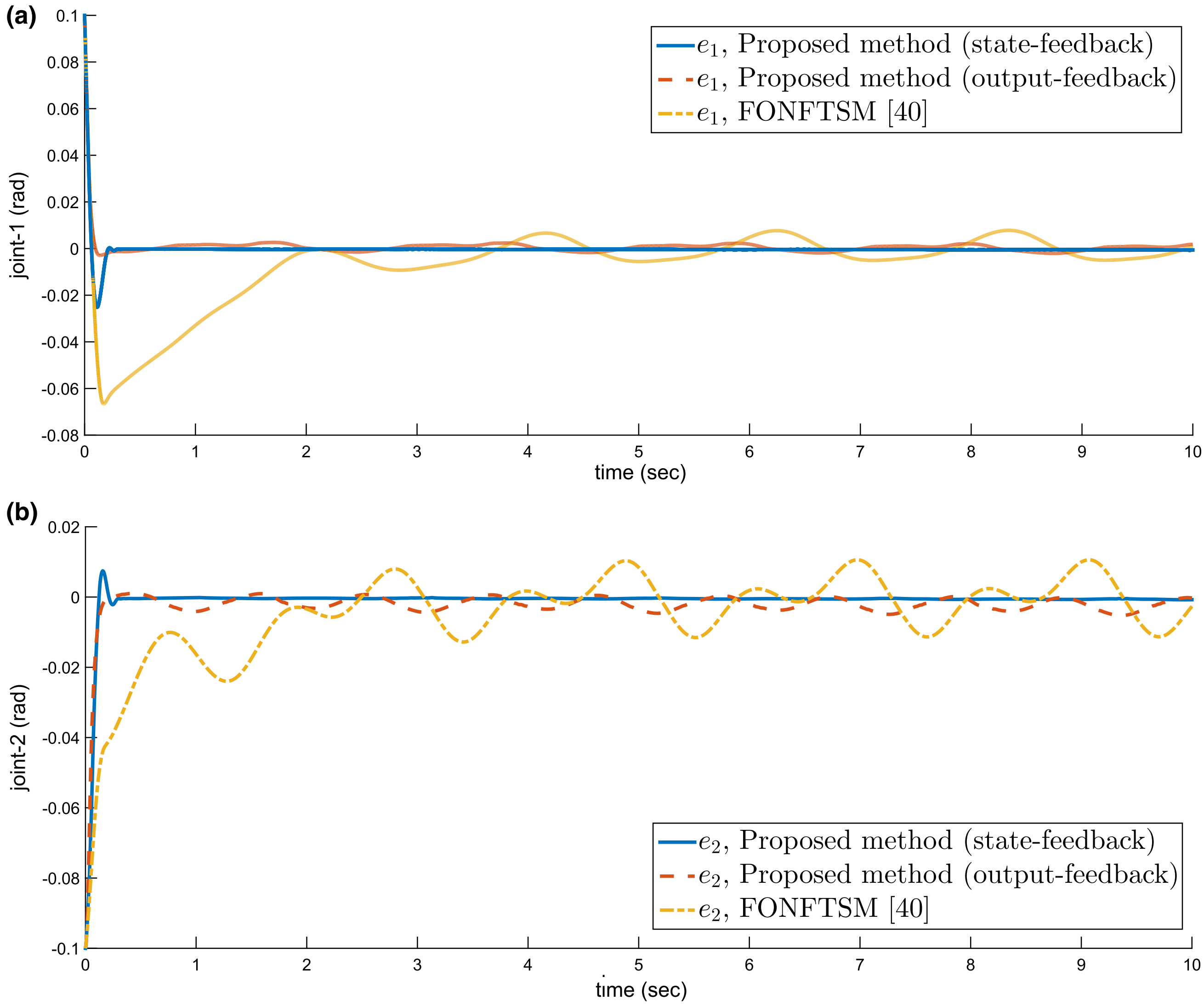

Fig. 4 Tracking error without measurement noise

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) + d = \tau \tag{41}$$

where

$$M(q) = \begin{bmatrix} (m_1+m_2)l_1^2 + m_2 l_2^2 + 2m_2 l_1 l_2 \cos(q2) + I_1 & m_2 l_2^2 + m_2 l_1 l_2 \cos(q_2) \\ m_2 l_2^2 + m_2 l_1 l_2 \cos(q_2) & m_2 l_2^2 + I_2 \end{bmatrix},$$
$$C(q,\dot{q}) = \begin{bmatrix} -m_2 l_1 l_2 \sin(q_2)\dot{q}_1 - 2m_2 l_1 l_2 \sin(q_2)\dot{q}_2 \\ m_2 l_1 l_2 \sin(q_2)\dot{q}_2 \end{bmatrix},$$
$$G(q) = \begin{bmatrix} (m_1+m_2) l_1 g \cos(q_2) + m_2 l_2 g \cos(q_1+q_2) \\ m_2 l_2 g \cos(q_1+q_2) \end{bmatrix},$$
$$d = \begin{bmatrix} 0.5\dot{q}_1 + sin(3q_1) \\ 1.3\dot{q}_2 - sin(2q_2) \end{bmatrix},$$

and

$$q_d = \begin{bmatrix} 0.5\sin(3t) + 0.05\sin(6t) \\ -0.2\cos(3t) - 0.04\sin(6t) - 0.2\cos(6t) + 0.4 \end{bmatrix}$$

is selected as the desired trajectory (Table 1).

To demonstrate the effectiveness of the proposed method, suitable parameters are selected as follows: for sliding surface (12), $\bar{k}_1 = diag(0.1, 0.1)$, $\bar{k}_2 = diag(60, 60)$ and $\upsilon = 0.8$. Moreover, control torque (30) and adaptive laws (34) parameters are chosen as follows, $N = diag(0.1, 0.1)$, $k_1 = diag(20, 20)$, $k_2 = diag(10, 10)$, $\zeta = 2$, $\gamma = 0.01$. And the initial positions of adaptive laws and angular positions are selected as $\hat{\varsigma}(0) = (2, 2, 2)$, $q_1(0) = 0.1$ and $q_2(0) = -0.3$, respectively. Furthermore, the FONFTSM sliding manifold, adaptation laws, and control input are taken from Ahmed et al. (2018b). And the FONFTSM control parameters are selected as same as the proposed control method.

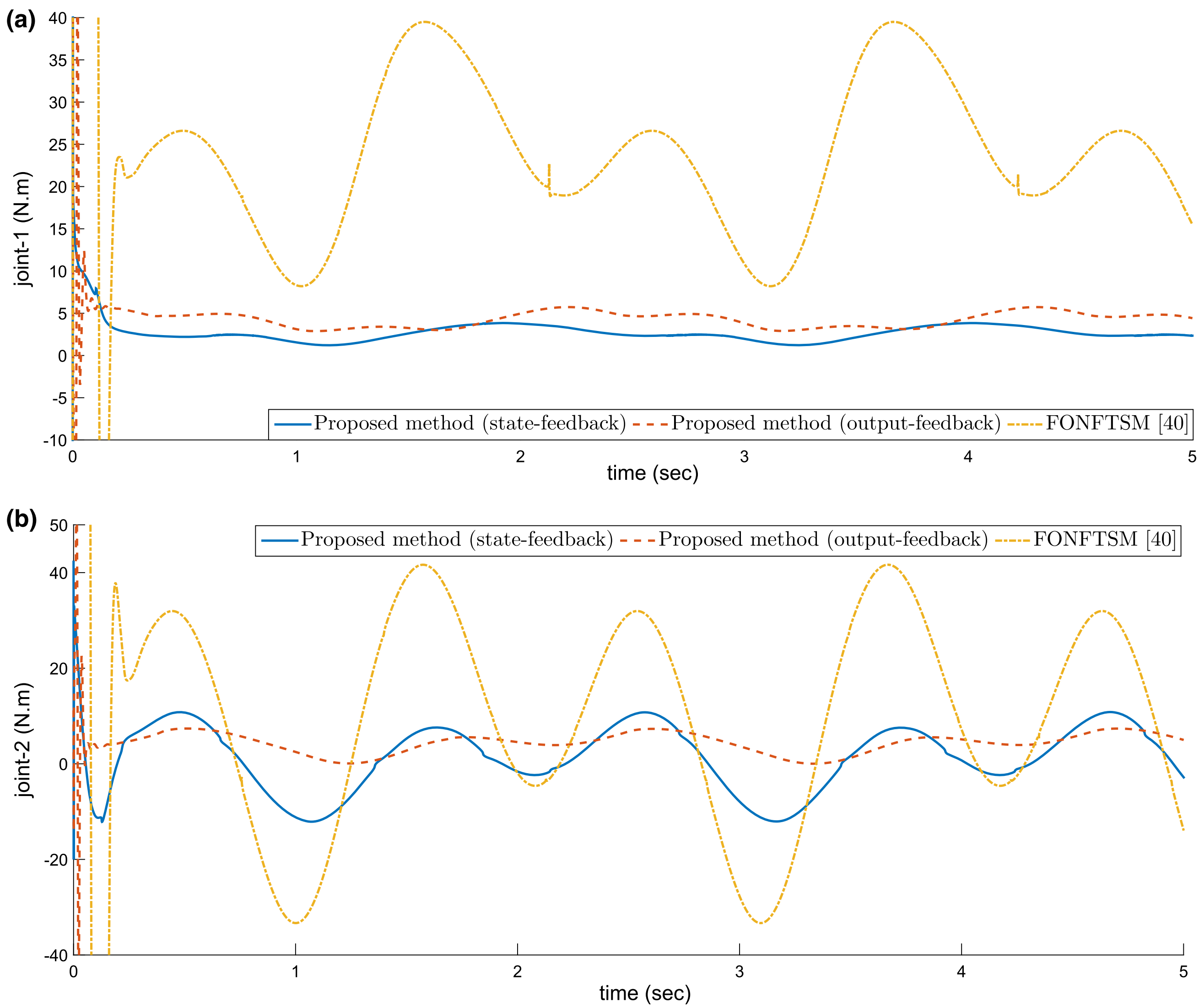

Fig. 5 Control inputs without measurement noise

5.2 Comparative Simulation Results Between Proposed SF and OF Schemes

The simulation results have been tested by Matlab/Simulink software using Runge–Kutta solver with 0.001s fixed-step size, and 'FOMCON toolbox' has been used for fractional calculus (Tepljakov et al. 2011). To find the best FO value of the proposed method, the trail-&-error method is used between the defined range $0<\beta<1$. Thus, for the selection of appropriate β, the corresponding values of $\beta =$ 0.1, 0.5 and 0.99 are selected. Hence, the best performance of joint position trajectory tracking is obtained at $\beta = 0.99$, and the simulation result of tracking error at different value of β is depicted in Fig. 2.

After the selection of suitable value of β, the comparative results are presented between the SF and OF control of proposed method with FONFTSM. Thus, the parameters of the controller are fairly selected. In this subsection, the simulation results are made in the absence of measurement noise. Therefore, to demonstrate the performance of the proposed method with FONFTSM, their corresponding simulation results are shown in Figs. 3, 4 and 5, which demonstrate, respectively, their compared position tracking, tracking error and control inputs signals.

From these illustrative results, it is clearly concluded that the trajectory tracking performance of the proposed SF and OF control methods has better convergence speed and chatter-free control performances than FONFTSM.

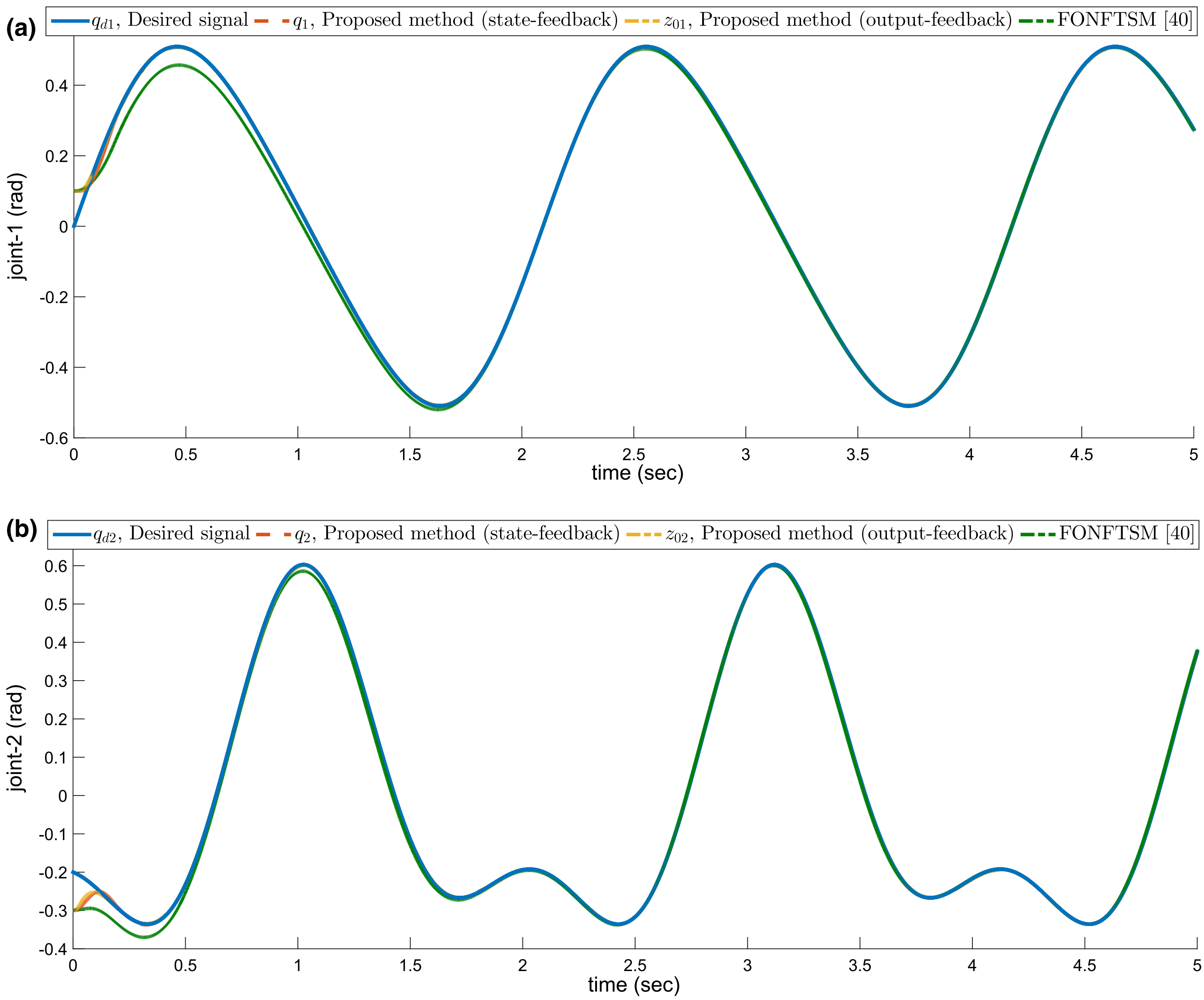

Fig. 6 Position tracking in the presence of measurement noise

5.3 Comparative Simulation Results Between Proposed SF and OF Schemes with FONFTSM Under Measurement Noise

In practical applications, the measurement noise is usually unavoidable, thus the effect of noise is introduced using a band-limited white noise module into the joint position of the robotic manipulators and the noise power is chosen as 4×10^{-8} with sample time 0.001 (Wang et al. 2020b).

In this subsection, the performance of the SF and OF of the proposed AOFSTSM method in comparison with FONFTSM in the presence of external disturbances and measurement noise is further demonstrated. Therefore, their corresponding simulation results of position tracking, tracking error and control torque input are demonstrated, respectively, in Figs. 6, 7 and 8.

From simulations (Figs. 6, 7), it can be observed that the performances regarding position tracking in terms of convergence speed and trajectory tracking precision are excellent. However, Fig. 8 shows that the small chattering in the control inputs of proposed method due to noise, while the compared FONFTSM method has immense amount of oscillations. Hence, the overall illustrated simulation performance confirms the effectiveness of the proposed scheme to unknown dynamics under external disturbances and measurement noise, and it obtains high tracking performances in terms of fast convergence, small steady state error and control inputs.

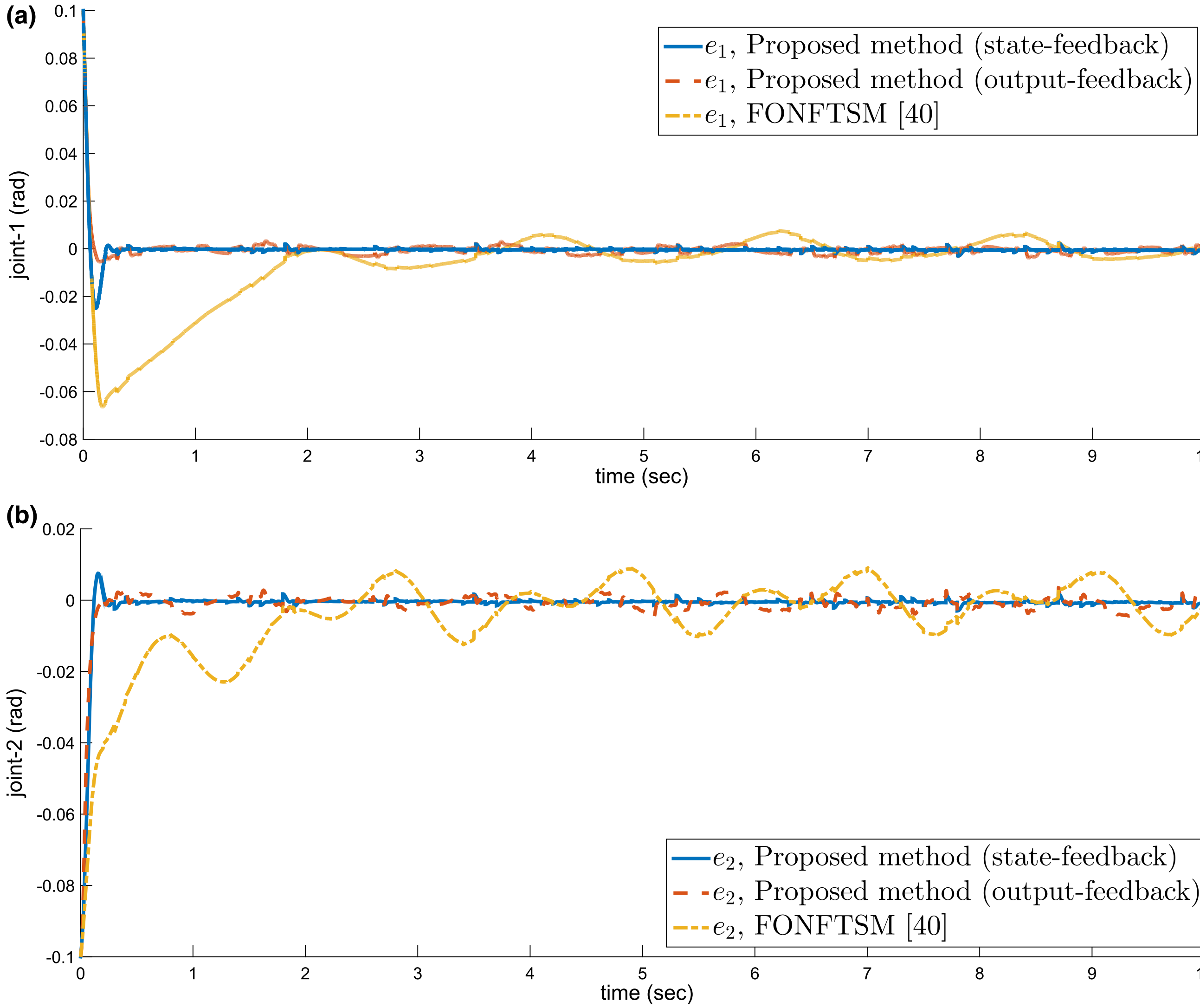

Fig. 7 Tracking error in the presence of measurement noise

6 Conclusion

In the presented paper, adaptive fractional super-twisting sliding mode control (AOFSTSM) is proposed to control the uncertain dynamics of robotic manipulator in the presence of external disturbances and measurement noise. The proposed model-independent control method is designed to improve the performance, and it provides robustness and easier implementation without knowing the knowledge of system dynamics. On the other hand, the finite-time stability investigation of the closed-loop system is established by the Lyapunov criterion. To exemplify the robustness and effectiveness of the proposed AOFSTSM method, an uncertain 2-DOF robotic manipulator with external disturbances and noise is used. Thus, from their corresponding simulation results, it is easily noting that the proposed method in comparison with FONFTSM has better performances in terms of robustness, fast response and tracking error.

This work presents the control design of unknown dynamics of robotic manipulator by model-free adaptive control method. Therefore, the future research work suggests that the adaptive based controller can be designed a robust controller under non-smooth nonlinearities such as saturation and dead-zone.

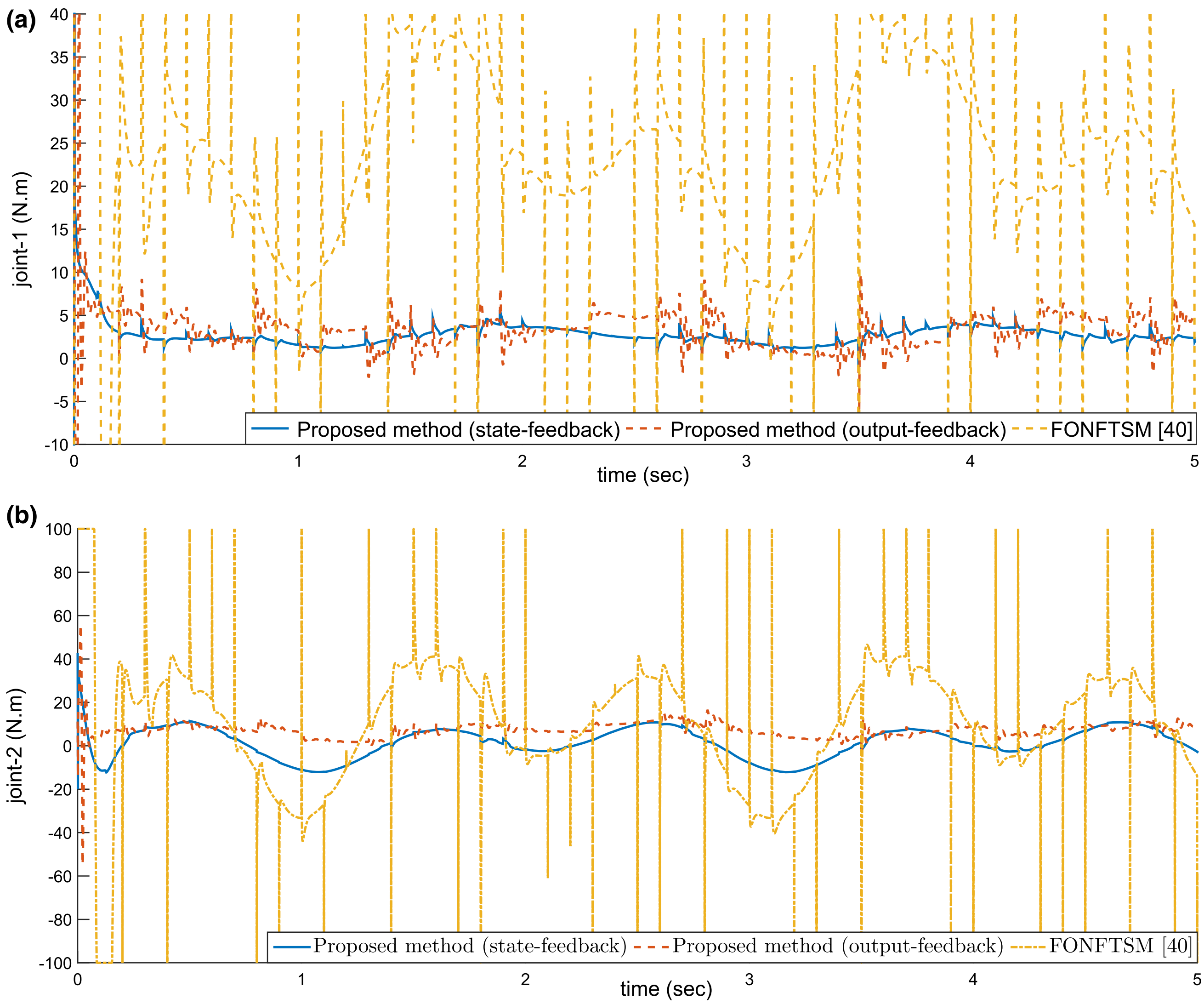

Fig. 8 Control inputs in the presence of measurement noise

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