(c) The system temperature is changed to a new value T_2 such that $T_2 = 2T_1$. Quantify the effect of this change on the probability determined in part (b) (i.e. By what factor will the probability determined in part (b) change because of this temperature change?).

New, $T_2 = 2T_1$

$$(P_A)_{T_2} = e^{\frac{4E_A}{KT_2}}$$
$$= e^{\frac{4E_A}{K2T_1}}$$
$$= e^{\frac{1}{2}\cdot\frac{4E_A}{KT_1}}$$
$$= e^{\frac{1}{2}}\cdot e^{\frac{4E_A}{KT_1}}$$
$$= e^{1/2}\cdot(P_A)_{T_1}$$

$(P_A)_{T_1}$ will change by the factor of $e^{-1/2}$.

because,
$$(P_A)_{T_1} = \frac{(P_A)_{T_2}}{e^{1/2}}$$
$$= (P_A)_{T_2}\, e^{-1/2}$$

22 points

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2. For the grand canonical ensemble, (μ, V, T) consider fluctuations in the number of particles. The relative magnitude of these fluctuations in the number of particles is given by the expression:

$$\frac{\sigma_N}{\overline{N}} = \left(\frac{k_B\,T\,\kappa}{V}\right)^{1/2}$$

where $\overline{N}$ is the average number of particles, k_B is the Boltzmann constant, T is the temperature, κ is the isothermal compressibility, and V is the volume of the system.

$k = -\frac{1}{V}\left(\frac{\delta V}{\delta P}\right)$

(a) Derive an expression for the relative fluctuations in the number of particles in an ideal gas system (**Hint:** Use the ideal gas equation of state).

$\Rightarrow \frac{\delta P}{\delta V} = -\frac{1}{KV}$

For ideal gas system, $PV = NKT$

$$\Rightarrow V = \frac{NKT}{P}$$
$$\Rightarrow \frac{dV}{dP} = -\frac{NKT}{P^2}$$
$$= -\frac{PV}{P^2} = -\frac{V}{P}$$

For isothermal compressibility, we know that,

$$k = -\frac{1}{V}\cdot\left(\frac{\delta V}{\delta P}\right)_{N,T}$$
$$= -\frac{1}{V}\cdot\left(-\frac{V}{P}\right) \qquad \left[\because \frac{dV}{dP} = -\frac{V}{P} \Rightarrow \frac{\delta V}{\delta P} = -\frac{V}{P}\right]$$
$$= \frac{1}{P}$$