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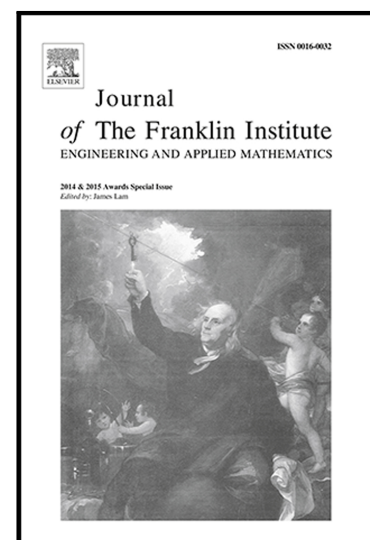
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Distributed estimation and control for two-target tracking mobile sensor networks

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Abstract

In this paper, we investigate a new estimation and control strategy for two-target tracking mobile sensor networks. Different from the single-target tracking problem, the two-target tracking one has to consider the interaction between followers in different groups. Based on a new flocking control algorithm and distributed filter, all mobile sensors can split into two groups to track their own target and form a cohesive flock with their neighbors. Stability analysis is conducted based on cascading Lyapunov method and matrix theory. Furthermore, a sufficient condition for the convergence is given in the form of the boundary conditions of feedback gains. Finally, a numerical example is presented to illustrate the validity of the proposed theoretical results.

Keywords: Distributed filter, flocking, two-target tracking, mobile sensor network

1. Introduction

Recently, due to the flexibility of deployment and low cost, sensor networks have appealed to researchers and practitioners in many areas, which is widely applied in surveillance systems, target tracking, information processing and so on [1], [2], [3]. Each sensor in mobile sensor networks can process information, communicate with its neighbors and move with the target. Generally speaking, every sensor uses a distributed estimation algorithm to estimate the uncertainty target and utilizes a control algorithm to move close to the target.

As an important part of sensor networks, filtering algorithms have been widely studied [4], [5], [6]. In particular, consensus algorithms have been proved to be effective tools for performing distributed computation tasks [7]. Inspired by consensus algorithms in multi-agent systems [8], [9], distributed Kalman filters have been proposed, which can help the estimation value of every sensor

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reach consensus effectively and reduce the mean square error of the estimation [10], [11], [12].

Compared with static sensor networks, the mobile sensor network has better mobility and better performance, which is more attractive to researchers. In [13], a novel theoretical framework is introduced for mobile sensor networks, and sensors based on a flocking algorithm move with a single target. Flocking portrays the collective behavior derived from the interacting individuals using only limited local information [14]. In [15], Reynolds introduced three rules for flocking: 1) flocking centering, 2) collision avoidance, 3) velocity matching. In Olfati-saber's work, the estimation of the target that is processed by a distributed filtering algorithm is directly used as the feedback term of the flocking algorithm. By applying this cascade estimation and control algorithm, followers construct a mobile sensor network. In [16], mobile sensor networks have been investigated with time delay. Mobile sensor networks with a single target have been used in large-scale surveillance systems [17]. However, compared with the single-target case, the multiple-target tracking problem is more valuable in the surveillance systems. In fact, many scenarios in real life have multiple-target tracking problems. In [18], multi-target problem has been studied for the flocking of multi-agent systems. It should be pointed out that there may exist interaction between the targets when they are close to each other, which motivates the researches focus on the coupled multiple-target tracking problems [19], [20].

According to the above discussion, flocking-based mobile sensor networks with two coupled targets will be considered in this paper. Generally speaking, this paper extends static sensor networks with two coupled targets [20] to the case of mobile sensor network and extends the mobile sensor network with one target [13] to the case with two coupled targets. To the best of our knowledge, this problem does not attract enough attention in spite of its evident importance in application. Therefore, this paper intends to fill such a gap. It should be pointed out that the interaction between two sensors in different groups may lead the system to be unstable, and how to use local information to design an effective algorithm is a crucial problem. Overall, the main contribution of this paper is to design a new flocking control algorithm to achieve two-target tracking and combine it with a distributed filter to construct a mobile sensor network to estimate the targets. By applying the proposed control and estimation strategy, mobile sensors can form flock with their neighbors in the same group and track their own target successfully.

This paper is organized as follows: In Section II, the mathematical model of this problem will be introduced. In Section III, the main result will be given and several definitions and assumptions will be presented. In Section VI, by constructing a Lyapunov function, the stability of this system will be proved. In Section V, a numerical example will be presented to illustrate the validity of the proposed flocking algorithm. Finally, conclusions are drawn.

Notations: In this paper, ∇f is the gradient of the function f , and $\mathbf{0}$ is the zero matrix. $\lambda_{\max}(A)$ represents the maximum eigenvalue of matrix A . $\|\cdot\|$ means the Euclidean norm and $\otimes$ refers to the Kronecker product.

2. Model Description

Consider two targets, labeled as α and β moving in an n dimensional Euclidean space, with dynamics as follows:

$$\begin{cases} \dot{q}_\gamma^\alpha(t) = p_\gamma^\alpha(t), \\ \dot{p}_\gamma^\alpha(t) = B_\alpha f(p_\gamma^\alpha(t), q_\gamma^\alpha(t)) + C_\alpha p_\gamma^\beta(t), \\ \dot{q}_\gamma^\beta(t) = p_\gamma^\beta(t), \\ \dot{p}_\gamma^\beta(t) = B_\beta f(p_\gamma^\beta(t), q_\gamma^\beta(t)) + C_\beta p_\gamma^\alpha(t), \end{cases} \quad (1)$$

where $q_\gamma^\alpha, p_\gamma^\alpha, q_\gamma^\beta, p_\gamma^\beta \in R^n$ are the states of targets. $B_\alpha, B_\beta \in R^{n \times n}$. The nonlinear function $f : R^+ \times R^n \rightarrow R^n$. C_α and C_β are two matrices which represent the interaction between two targets.

Suppose there are $N + M$ followers splitting into two groups: α -group which measures the state of target α and β -group which measures the state of target β . Without loss of generality, let the first N followers be in α -group and the remaining followers be in β -group, then two sets can be represented as $S_\alpha = \{1, 2, \dots, N\}$, $S_\beta = \{N + 1, N + 2, \dots, N + M\}$. The signals, which the follower sensors receive, can be described as follows:

$$\begin{cases} z_i^\alpha(t) = D_i^\alpha s_\alpha(t) + H_\alpha v_i^\alpha(t), & i \in S_\alpha, \\ z_j^\beta(t) = D_j^\beta s_\beta(t) + H_\beta v_j^\beta(t), & j \in S_\beta, \end{cases} \quad (2)$$

where $s_\alpha(t) = [q_\gamma^\alpha, p_\gamma^\alpha] \in R^{2n}$, $s_\beta(t) = [q_\gamma^\beta, p_\gamma^\beta] \in R^{2n}$ are the states of the targets. And $z_i^\alpha \in R^l$ is the state received by the i th follower in α -group. $z_j^\beta \in R^l$ is the target state received by the j th follower in β -group. $D_i^\alpha, D_j^\beta \in R^{l \times 2n}$ are two output matrices of i th and j th follower, respectively. $v_i^\alpha(t) \in R^n$ and $v_j^\beta(t) \in R^n$ are zero-mean Gaussian noises. $H_\alpha, H_\beta \in R^{l \times n}$ denote the strengths of the noises.

All the sensors can use the filtering algorithm mentioned in [20] to estimate the states of the targets, which is presented as follows:

Consider there are two kinds of sensors in each group. The first type of sensors is more powerful, which can broadcast its information to all sensors. And the second type of sensors' communication ability is limited, which only communicate with the neighbors. Without loss of generality, let the first sensor in each group be the powerful one. The distributed filtering protocol is designed

85 as follows:

$$\begin{aligned}
 \dot{x}_i(t) &= B_1 F(x_i(t)) + \sum_{j \in S_\alpha} g_{ij} [x_j(t) - x_i(t)] + C_1 [x_{N+1}(t)] \\
 &\quad + z_i^\alpha - D_i^\alpha x_i(t), \\
 \dot{y}_i(t) &= B_2 F(y_i(t)) + \sum_{j \in S_\beta} g_{ij} [y_j(t) - y_i(t)] + C_1 [x_1(t)] \\
 &\quad + z_i^\beta - D_i^\beta y_i(t),
 \end{aligned} \tag{3}$$

where $x_i(t) = [x_{i1}(t), x_{i2}(t), \dots, x_{i2n}(t)]^T = [\hat{q}_\gamma^{\alpha T}, \hat{p}_\gamma^{\alpha T}]^T$ is the estimation of target state $s_\alpha(t)$ by the i th sensor in α -group and $y_i(t) = [y_{i1}(t), y_{i2}(t), \dots, y_{i2n}(t)]^T = [\hat{q}_\gamma^{\beta T}, \hat{p}_\gamma^{\beta T}]^T$ is the estimation of target state $s_\beta(t)$ by the i th sensor in β -group.

The sets of neighbors N_i^α, N_i^β is defined as:

$$\begin{aligned}
 N_i^\alpha &= \{i, j \in S_\alpha : \|q_i - q_j\| < r\}, \\
 N_i^\beta &= \{i, j \in S_\beta : \|q_i - q_j\| < r\},
 \end{aligned} \tag{4}$$

90 where $r > 0$ is the interaction range of every sensor. $G_\alpha = \{g_{ij}\}, i, j \in S_\alpha$ if $j \in N_i^\alpha$, $g_{ij} = 1$ else $g_{ij} = 0$ and $G_\beta = \{g_{ij}\}, i, j \in S_\beta$ if $j \in N_i^\beta$, $g_{ij} = 1$ else $g_{ij} = 0$. $B_1, B_2 \in R^{2n \times 2n}$ and $B_1 = \text{diag}\{I, B_\alpha\}$, $B_2 = \text{diag}\{I, B_\beta\}$. And $F(s_\alpha(t)) = [(p_\gamma^\alpha)^T, f(q_\gamma^\alpha, p_\gamma^\alpha)^T]^T \in R^{2n}$, $F(s_\beta(t)) = [(p_\gamma^\beta)^T, f(q_\gamma^\beta, p_\gamma^\beta)^T]^T \in R^{2n}$. In this way, C_1 and C_2 can be presented as follows:

$$C_1 = \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & C_\alpha \end{bmatrix}, C_2 = \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & C_\beta \end{bmatrix}. \tag{5}$$

95 Suppose each follower which tracks the state of targets moves with dynamics as follows:

$$\begin{cases} \dot{q}_i = p_i, \\ \dot{p}_i = u_i. \end{cases} \tag{6}$$

Each follower applies the following distributed control strategy:

$$\begin{aligned}
 u_i^\alpha &= -\nabla_{q_i} U(q_i) + f_{i,c} + f_i^\gamma, & i \in S_\alpha, \\
 u_i^\beta &= -\nabla_{q_i} U(q_i) + f_{i,c} + f_i^\gamma, & i \in S_\beta,
 \end{aligned} \tag{7}$$

where $U(q_i)$ is the collective potential function of the i th agent, which is defined the same as that in [21]. Figure 1 shows an example of this potential function. 100 The potential function is used to regulate position between the i th agent and its neighbors.

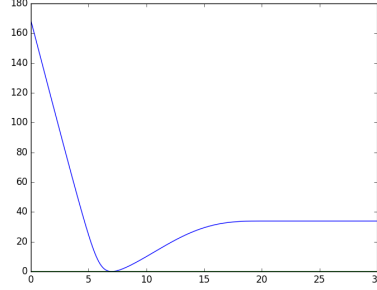


Figure 1: A smooth artificial potential function.

The second term $f_{i,c}$ is the velocity consensus term of the system:

$$\begin{aligned} f_{i,c} &= B_\alpha f(\hat{q}_{i,\gamma}, p_i) + C_\alpha p_{N+1} + \sum_{j \in S_\alpha} g_{ij}(p_j - p_i), & i \in S_\alpha, \\ f_{i,c} &= B_\beta f(\hat{q}_{i,\gamma}, p_i) + C_\beta p_1 + \sum_{j \in S_\beta} g_{ij}(p_j - p_i), & i \in S_\beta. \end{aligned} \quad (8)$$

The third term f_i^γ is a target tracking term which is designed as follows:

$$\begin{aligned} f_i^\gamma &= -c_1(q_i - \hat{q}_{i,\gamma}^\alpha) - c_2(p_i - \hat{p}_{i,\gamma}^\alpha), & i \in S_\alpha \\ f_i^\gamma &= -c_1(q_i - \hat{q}_{i,\gamma}^\beta) - c_2(p_i - \hat{p}_{i,\gamma}^\beta), & i \in S_\beta, \end{aligned} \quad (9)$$

where $\hat{q}_{i,\gamma}^\alpha, \hat{p}_{i,\gamma}^\alpha, \hat{q}_{i,\gamma}^\beta$ and $\hat{p}_{i,\gamma}^\beta$ are the estimations of target state by the i th sensor in α, β group respectively, and c_1, c_2 are feedback gains.

Since the estimation generated by (3) is directly used in the control input (7), this system is called a cascading distributed estimation and control system. The stability of the cascading mobile sensor networks requires the collision-free of mobile sensors and the tracking of mobile targets using a combination of the flocking and the filtering algorithms. In next section, we will give the stability analysis of this algorithm. Figure 2 shows the structure of the mobile sensor network.

3. Main Result

Definition 1. Give the definition that $L_\alpha = G_\alpha - \Delta(G_\alpha)$ and $L_\beta = G_\beta - \Delta(G_\beta)$, where $\Delta(G_\alpha)$ and $\Delta(G_\beta)$ represent the degree matrix of G_α, G_β respectively. Give the definition of $L_{\alpha\beta} = \{g_{ij}\}, i \in S_\alpha, j \in S_\beta, L_{\beta\alpha} = \{g_{ij}\}, i \in S_\beta, j \in S_\alpha$, are the communication matrices between α -group and β -group. According to the definitions the row sum of L_α, L_β is 0 and $L_{\alpha\beta}, L_{\beta\alpha}$ can be de-

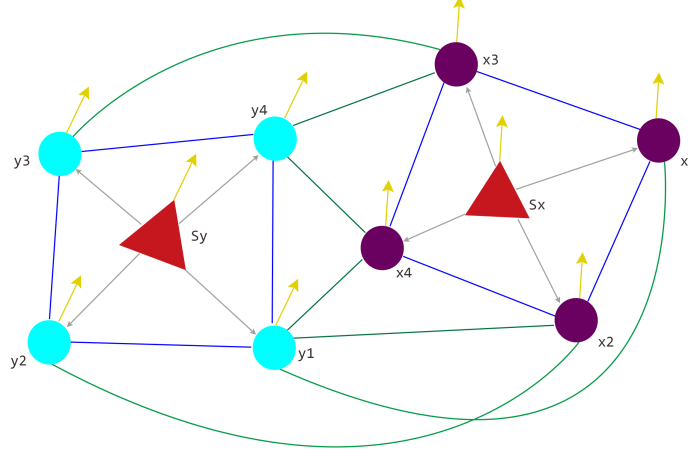


Figure 2: Mobile sensor network. The triangle represents targets and circle represents followers.

scribed as follows:

$$L_{\alpha\beta} = \begin{bmatrix} 1 & 0 & \dots & 0 \\ 1 & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 1 & 0 & \dots & 0 \end{bmatrix}, L_{\beta\alpha} = \begin{bmatrix} 1 & 0 & \dots & 0 \\ 1 & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 1 & 0 & \dots & 0 \end{bmatrix}. \quad (10)$$

Definition 2. [20] For an $N \times N$ irreducible square matrix H with nonnegative off-diagonal elements, the definition of $\beta(H)$ is showed as follows: H is decomposed uniquely as $H = P + F$, where F is a diagonal matrix and P is a zero row sum matrix. Let $\xi = [\xi_1, \xi_2, \dots, \xi_N]^T$ be the normalized left eigenvector P with respect to the eigenvalue zero satisfying $\sum_{i=1}^N \xi_i = 1$, and $\Xi = \text{diag}\{\xi_1, \xi_2, \dots, \xi_N\}$. Then, $\beta(H) = \lambda_{\max}(\Xi H + H^T \Xi)$.

Definition 3. [13] For a matrix $M \in R^{n \times m}$ and two vectors $x \in R^n$ and $y \in R^m$, the following inequation holds

$$2x^T M y \leq x^T x + y^T M^T M y.$$

Assumption 1. [22] There are two nonnegative constants ρ_1 and ρ_2 for the nonlinear function, such that:

$$\|f(x_2, y_2, t) - f(x_1, y_1, t)\| \leq \rho_1 \|x_2 - x_1\| + \rho_2 \|y_2 - y_1\|. \quad (11)$$

This is a Lipschitz-like condition, which is satisfied by many well known functions.

Lemma 1. For $A = [a_{ij}] \in R^{n \times n}$, let

$$R_i(A) = \sum_{j=1, j \neq i}^n |a_{ij}|, \quad i = 1, 2, \dots, n, \quad (12)$$

denote the absolute row sum of A without diagonal elements. All the eigenvalues of A are located in the set of n discs:

$$\bigcup_{i=1}^n \{z \in C : |z - a_{ii}| < R_i(A)\}. \quad (13)$$

Theorem 1. [20] Suppose that the nonlinear function satisfies Assumption 1. Consider a sensor network with a sensing model in (2), and each sensor applies the distributed estimation algorithm (3). When the noise is zero, the sensor network can track the targets $s_\alpha(t)$ and $s_\beta(t)$, if it satisfies the following LMI:

$$\Phi = \begin{bmatrix} \Psi_1 & \Xi^\alpha L_{\alpha\beta} + L_{\beta\alpha} \Xi^\beta \\ L_{\alpha\beta} \Xi^\alpha + \Xi^\beta L_{\beta\alpha} & \Psi_2 \end{bmatrix} < 0, \quad (14)$$

where $\Psi_1 = [\lambda_{\max}(B_\alpha B_\alpha^T) + \lambda_{\max}(L_\lambda^T L_\lambda) + \beta(\bar{L}_{\alpha\alpha})]I$ and $\Psi_2 = [\lambda_{\max}(B_\beta B_\beta^T) + \lambda_{\max}(L_\lambda^T L_\lambda) + \beta(\bar{L}_{\beta\beta})]I$. $\Xi^\alpha = \text{diag}\{\xi_1^\alpha, \xi_2^\alpha, \dots, \xi_n^\alpha\}$, $\Xi^\beta = \text{diag}\{\xi_1^\beta, \xi_2^\beta, \dots, \xi_n^\beta\}$. $\bar{L}_{\alpha\alpha} = L_{\alpha\alpha} - D^\alpha$, $\bar{L}_{\beta\beta} = L_{\beta\beta} - D^\beta$.

The dynamics of the estimation error $e_i^\alpha = x_i - s_\alpha$, $e_i^\beta = y_i - s_\beta$ can be presented as follows:

$$\begin{aligned} \dot{e}_i^\alpha &= B_\alpha f(x_i(t)) - B_\alpha f(s_\alpha(t)) + \sum_{j \in S_\alpha} L_{ij} e_j^\alpha(t) + \sum_{j \in S_\beta} L_{ij} e_j^\beta(t) - D_i^\alpha e_i^\alpha, i \in S_\alpha \\ \dot{e}_i^\beta &= B_\beta f(y_i(t)) - B_\beta f(s_\beta(t)) + \sum_{j \in S_\beta} L_{ij} e_j^\beta(t) + \sum_{j \in S_\alpha} L_{ij} e_j^\alpha(t) - D_i^\beta e_i^\beta, i \in S_\beta. \end{aligned} \quad (15)$$

Construct a Lyapunov function for this system :

$$V_e(t) = V_{e1}(t) + V_{e2}(t) = \sum_{i=1}^N e_i^{\alpha T} \varphi_i^\alpha e_i^\alpha + \sum_{i=1}^M e_i^{\beta T} \varphi_i^\beta e_i^\beta, \quad (16)$$

where $\varphi_i^\alpha = [\xi_{1i}^\alpha, \xi_{2i}^\alpha, \dots, \xi_{Ni}^\alpha]^T$, $\varphi_i^\beta = [\xi_{1i}^\beta, \xi_{2i}^\beta, \dots, \xi_{Ni}^\beta]^T$. Moreover, it can be proved that $\dot{V}(t) < 0$, if the LMI (14) is satisfied. Moreover, the conclusion that all estimators asymptotically reach a consensus can be reached. The detailed proof can be found in [20].

Theorem 2. Consider a mobile sensor network which is composed of $N + M$ sensors with dynamics (3) and each mobile sensor applies the estimation algo-

rithm (8). If the following condition is satisfied:

$$c_2 \geq \max\{\rho_1 \lambda_{\max}(B_\alpha), \rho_1 \lambda_{\max}(B_\beta)\} + \max\{\lambda_{\max}(C_\alpha) + \lambda_{\max}(C_\beta)\} + 1, \quad (17)$$

$$\begin{bmatrix} \Psi_1 + I_N \otimes P_1^T P_1 & \Xi^x L_{\alpha\beta} + L_{\beta\alpha} \Xi^y \\ L_{\alpha\beta} \Xi^x + \Xi^y L_{\beta\alpha} & \Psi_2 + I_M \otimes P_2^T P_2 \end{bmatrix} < 0, \quad (18)$$

where $P_1 = \begin{bmatrix} \rho_1 B_\beta + c_1 I & \mathbf{0} \\ \mathbf{0} & c_2 I \end{bmatrix}$, $P_2 = \begin{bmatrix} \rho_1 B_\alpha + c_1 I & \mathbf{0} \\ \mathbf{0} & c_2 I \end{bmatrix}$. And $\Psi_1, \Psi_2, \Xi^x, \Xi^y$ is defined the same as Theorem 1. ρ_1 and ρ_2 are the Lipschitz constants.

$\lambda_{\max}(A)$ refers to the biggest eigenvalue of A .

Then, the following statements hold:

- (1) The sensors form two cohesive flocks.
- (2) All estimators in the same group can asymptotically reach consensus on the state of the target.
- (3) The cascade system is stable.

Remark 1. Compared with Theorem 1, Theorem 2 extends the static sensor network to the mobile case. The range of the feedback gains is given to make sure the system is stable when there exists a dynamical error.

4. Stability analysis

In this section, the detailed proof of Theorem 2 will be presented.

The dynamics of followers in α -group can be written as follows:

$$\begin{cases} \dot{q}_i^\alpha = p_i^\alpha, \\ \dot{p}_i^\alpha = -\nabla_{q_i} U(q_i^\alpha) + f_{i,c} + f_i^\gamma, \quad i \in S_\alpha. \end{cases} \quad (19)$$

Let $\zeta_i = q_i - q_i^\gamma$ and $\xi_i = p_i - p_i^\gamma$, and the cascade system can be rewritten as follows:

$$\Sigma : \begin{cases} \dot{\zeta}_i^\alpha = \xi_i^\alpha, \\ \dot{\xi}_i^\alpha = -\nabla_{\zeta_i} U(\zeta_i^\alpha) + f_{i,c} + f_i^\gamma, \quad i \in S_\alpha, \end{cases} \quad (20)$$

$$\begin{aligned} \Sigma_e : \dot{e}_i^\alpha &= B_\alpha f(x_i(t)) - B_\alpha f(s_\alpha(t)) + \sum_{j \in S_\alpha} L_{ij} e_j^\alpha(t) \\ &+ \sum_{j \in S_\beta} L_{ij} e_j^\beta(t) - D_i^\alpha e_i^\alpha, \quad i \in S_\alpha. \end{aligned} \quad (21)$$

And the dynamics of the agent in β -group can be written as follows:

$$\Sigma : \begin{cases} \dot{\zeta}_i^\beta = \xi_i^\beta, \\ \dot{\xi}_i^\beta = -\nabla_{\zeta_i} U(\zeta_i^\beta) + f_{i,c} + f_i^\gamma, \quad i \in S_\beta, \end{cases} \quad (22)$$

$$\begin{aligned} \Sigma_e : \dot{e}_i^\beta &= B_\beta f(y_i(t)) - B_\beta f(s_\beta(t)) + \sum_{j \in S_\beta} L_{ij} e_j^\beta(t) \\ &+ \sum_{j \in S_\alpha} L_{ij} e_j^\alpha(t) - D_i^\beta e_i^\beta, \quad i \in S_\beta. \end{aligned} \quad (23)$$

Consider a Lyapunov function for this cascade system:

$$V(\zeta, \xi, t) = \sum_{i \in S_\alpha \cup S_\beta} H_i(\zeta_i) + \sum_{i \in S_\alpha} \frac{1}{2} \xi_i^T \xi_i + \sum_{i \in S_\beta} \frac{1}{2} \xi_i^T \xi_i + V_e(t), \quad (24)$$

where $H_i(\zeta_i) = U_i(\zeta_i) + \frac{1}{2} c_1 \|q_i - q_\gamma\|^2, i \in S_\alpha \cup S_\beta$.

By directly differentiating, we obtain:

$$\dot{V}(\zeta, \xi, t) = \sum_{i \in S_\alpha \cup S_\beta} \dot{H}_i(\zeta_i) + \sum_{i \in S_\alpha} \xi_i^T \dot{\xi}_i + \sum_{i \in S_\beta} \xi_i^T \dot{\xi}_i. \quad (25)$$

Let the definition of φ_i in α -group be:

$$\begin{aligned} \varphi_i &= \xi_i^T \dot{\xi}_i \\ &= \xi_i^T (-\nabla_{\zeta_i} U(\zeta_i^\alpha) + f_{i,c} + f_{i,\alpha}^\gamma), \quad i \in S_\alpha, \end{aligned} \quad (26)$$

170 which can be separated into three parts:

$$\begin{aligned} \varphi_{i1} &= -\xi_i^T \nabla_{\zeta_i} U(\zeta_i), \\ \varphi_{i2} &= \xi_i^T (B_\alpha f(\hat{q}_{i,\gamma}, p_i) - f(q_i^\gamma, p_i^\gamma) + C_\alpha \xi_{N+1} + \sum_{j \in S_\alpha} g_{ij}(\xi_j - \xi_i)), \\ \varphi_{i3} &= \xi_i^T (-c_1(q_i - \hat{q}_{i,\gamma}^\alpha) - c_2(p_i - \hat{p}_{i,\gamma}^\alpha)). \end{aligned} \quad (27)$$

According to (11), Φ_{i2} can be rewritten as follows:

$$\begin{aligned} \varphi_{i2} &\leq \xi_i^T (B_\alpha (\rho_1(\hat{q}_{i,\gamma} - q_i^\gamma) + \rho_2(p_i - p_i^\gamma)) + C_\alpha \xi_{N+1} \\ &+ \sum_{j \in S_\alpha} g_{ij}(\xi_j - \xi_i)) \\ &\leq \xi_i^T (B_\alpha (\rho_1 e_{i1} + \rho_2 \xi_i) + C_\alpha \xi_{N+1} + \sum_{j \in S_\alpha} g_{ij}(\xi_j - \xi_i)), \end{aligned} \quad (28)$$

where $e_i = x_i - s_\alpha = [e_{i1}^T, e_{i2}^T]^T \in R^{2n}, i \in S_\alpha$, which $e_{i1}^\alpha, e_{i2}^\alpha \in R^n$.

The third part of φ_i can be written as:

$$\begin{aligned}\varphi_{i3} &= \xi_i^T (-c_1(q_i - q_i^\gamma + q_i^\gamma - \hat{q}_{i,\gamma}^\alpha) - c_2(p_i - p_i^\gamma + p_i^\gamma - \hat{p}_{i,\gamma}^\alpha)) \\ &= \xi_i^T (-c_1\zeta_i - c_2\xi_i + c_1e_{i1} + c_2e_{i2}).\end{aligned}\quad (29)$$

Combine this three parts, then the following can be obtained:

$$\begin{aligned}\varphi_i &\leq -\xi_i^T \nabla_{\zeta_i} U(\zeta_i) + \xi_i^T (B_\alpha(\rho_1 e_{i1} + \rho_2 \xi_i) + C_\alpha \xi_{N+1} \\ &\quad + \sum_{j \in S_\alpha} g_{ij}(\xi_j - \xi_i)) + \xi_i^T (-c_1\zeta_i - c_2\xi_i + c_1e_{i1} + c_2e_{i2}) \\ &= -\xi_i^T \nabla_{\zeta_i} U(\zeta_i^\alpha) - c_1\xi_i^T \zeta_i + \xi_i^T (B_\alpha \cdot \rho_2 I \xi_i + \sum_{j \in S_\alpha} g_{ij}(\xi_j - \xi_i)) \\ &\quad - c_2\xi_i^T \xi_i + \xi_i^T C_\alpha \xi_{N+1} + \xi_i^T (B_\alpha \cdot \rho_1 I e_{i1} + c_1e_{i1} + c_2e_{i2}).\end{aligned}\quad (30)$$

175 For all the sensors in β group, φ_i can be obtained in a manner:

$$\begin{aligned}\varphi_i &= -\xi_i^T \nabla_{\zeta_i} U(\zeta_i) - c_1\xi_i^T \zeta_i + \xi_i^T (B_\beta \cdot \rho_2 I \xi_i + \sum_{j \in S_\beta} g_{ij}(\xi_j - \xi_i)) \\ &\quad - c_2\xi_i^T \xi_i + \xi_i^T C_\beta \xi_1 + \xi_i^T (B_\beta \cdot \rho_1 I e_{i1} + c_1e_{i1} + c_2e_{i2}), \quad i \in S_\beta,\end{aligned}\quad (31)$$

where $e_i = y_i - s_\beta = [e_{i1}^T, e_{i2}^T]^T \in R^{2n}$, $i \in S_\beta$, which $e_{i1}, e_{i2} \in R^n$.
Now, calculate the Lypunov function $V(t)$

$$\begin{aligned}V(t) &\leq \sum_{i \in S_\alpha \cup S_\beta} \xi_i^T (\nabla_{\zeta_i} U(\zeta_i) + c_1\zeta_i) + V_e(t) \\ &\quad + \sum_{i \in S_\alpha} \xi_i^T (-\nabla_{\zeta_i} U(\zeta_i) - c_1\zeta_i + B_\alpha \cdot \rho_2 I \xi_i + \sum_{j \in S_\alpha} g_{ij}(\xi_j - \xi_i) \\ &\quad - c_2\xi_i + C_\alpha \xi_{N+1} + B_\alpha \cdot \rho_1 I e_{i1} + c_1e_{i1} + c_2e_{i2}) \\ &\quad + \sum_{i \in S_\beta} \xi_i^T (-\nabla_{\zeta_i} U(\zeta_i) - c_1\zeta_i + B_\beta \cdot \rho_2 I \xi_i + \sum_{j \in S_\beta} g_{ij}(\xi_j - \xi_i) \\ &\quad - c_2\xi_i + C_\beta \xi_1 + B_\beta \cdot \rho_1 I e_{i1} + c_1e_{i1} + c_2e_{i2}) \\ &= \sum_{i \in S_\alpha} \xi_i^T (B_\alpha \cdot \rho_2 I \xi_i + \sum_{j \in S_\alpha} g_{ij}(\xi_j - \xi_i) - c_2\xi_i + C_\alpha \xi_{N+1} \\ &\quad + B_\alpha \cdot \rho_1 I e_{i1} + c_1e_{i1} + c_2e_{i2}) \\ &\quad + \sum_{i \in S_\beta} \xi_i^T (B_\beta \cdot \rho_2 I \xi_i + \sum_{j \in S_\beta} g_{ij}(\xi_j - \xi_i) - c_2\xi_i + C_\beta \xi_1 \\ &\quad + B_\beta \cdot \rho_1 I e_{i1} + c_1e_{i1} + c_2e_{i2}) + V_e(t).\end{aligned}\quad (32)$$

Then, $V(t)$ can be written as follows:

$$\begin{aligned}
 V(t) &\leq \xi_\alpha^T (I_N \otimes \rho_1 B_\alpha \xi_\alpha + L_\alpha \otimes I_n \xi_\alpha - c_2 \xi_\alpha + L_{\alpha\beta} \otimes C_\alpha \xi_\beta) \\
 &\quad + \xi_\beta^T (I_M \otimes \rho_1 B_\beta \xi_\beta + L_\beta \otimes I_n \xi_\beta - c_2 \xi_\beta + L_{\beta\alpha} \otimes C_\beta \xi_\alpha) \\
 &\quad + \sum_{i \in S_\alpha} \xi_i^T ((\rho_1 B_\beta + c_1 I) e_{i1} + c_2 e_{i2}) \\
 &\quad + \sum_{i \in S_\beta} \xi_i^T ((\rho_1 B_\alpha + c_1 I) e_{i1} + c_2 e_{i2}) + V_e(t) \\
 &\leq \xi_\alpha^T ((\rho_1 \lambda_{\max}(B_\alpha) - c_2) I_{nN} \xi_\alpha + \lambda_{\max}(C_\alpha) L_{\alpha\beta} \otimes I_n \xi_\beta) \\
 &\quad + \xi_\beta^T ((\rho_1 \lambda_{\max}(B_\beta) - c_2) I_{nM} \xi_\beta + \lambda_{\max}(C_\beta) L_{\beta\alpha} \otimes I_n \xi_\alpha) \\
 &\quad + \sum_{i \in S_\alpha} \xi_i^T \xi_i + \sum_{i \in S_\beta} \xi_i^T \xi_i + \sum_{i \in S_\alpha} e_i^T P_1^T P_1 e_i + \sum_{i \in S_\beta} e_i^T P_2^T P_2 e_i + V_e(t) \\
 &\leq \xi^T \psi_1 \xi^T + \sum_{i \in S_\alpha} e_i^T P_1^T P_1 e_i + \sum_{i \in S_\beta} e_i^T P_2^T P_2 e_i + V_e(t) \\
 &\leq \xi^T \psi_1 \xi^T + e \psi_2 e,
 \end{aligned} \tag{33}$$

where $\xi = [\xi_1^T, \xi_2^T, \dots, \xi_{N+M}^T]^T$, $e = [e_1^T, e_2^T, \dots, e_{N+M}^T]^T$, P_1, P_2 and ψ_2 are defined in Theorem 2 and ψ_1 is shown as follows:

$$\psi_1 = \begin{bmatrix} (\rho_1 \lambda_{\max}(B_\alpha) + 1 - c_2) I_{nN} & \lambda_{\max}(C_\alpha) L_{\alpha\beta} \otimes I_n \\ \lambda_{\max}(C_\beta) L_{\beta\alpha} \otimes I_n & (\rho_1 \lambda_{\max}(B_\beta) + 1 - c_2) I_{nM} \end{bmatrix}, \tag{34}$$

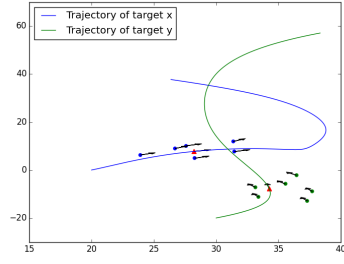
The row sum of ψ_1 is less than zero, if $c_2 \geq \max\{\rho_1 \lambda_{\max}(B_\alpha), \rho_1 \lambda_{\max}(B_\beta)\} + \max\{\lambda_{\max}(C_\alpha) + \lambda_{\max}(C_\beta)\} + 1$. The matrix can be decomposed into two matrices H_1 and H_2 . Since the row sum of H_1 is zero and all the diagonal elements are negative according to Lemma 1, H_i is negative semi-definite, and H_2 is a diagonal matrix with semi-negative elements, $\psi < 0$ is satisfied when $c_2 \geq \max\{\rho_1 \lambda_{\max}(B_\alpha), \rho_1 \lambda_{\max}(B_\beta)\} + \max\{\lambda_{\max}(C_\alpha) + \lambda_{\max}(C_\beta)\} + 1$. This completes the proof. $\square$

5. Numerical Example

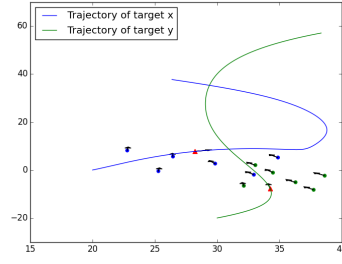
In this section, a numerical example will be presented to illustrate the validity of the proposed estimation and control strategy. Consider a mobile sensor network with twelve agents moving on a plane. There are six agents in α -group, and six agents in β -group. The dynamical parameters of the targets are:

$$B_\alpha = \begin{bmatrix} 0 & 0.5 \\ 0.8 & 0 \end{bmatrix}, B_\beta = \begin{bmatrix} 0 & -0.3 \\ 0.1 & 0 \end{bmatrix}. \tag{35}$$

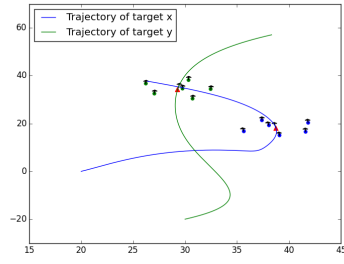
The nonlinear function f can be described as $f: f(x) = [\cos(x_1), \sin(x_2)]^T$. This function can satisfy Assumption 1. So the Lipschitz constants $\rho_1 = 1$ and $\rho_2 = 1$. The communication distance is 20. If the distance between two followers is



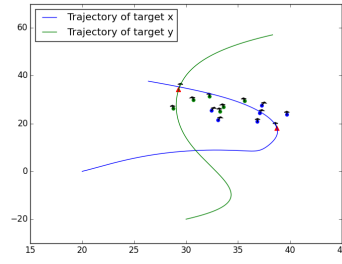
(a)



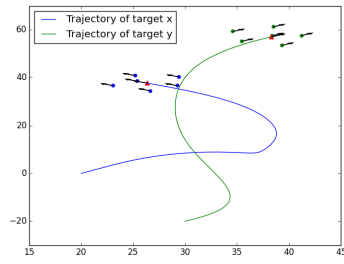
(b)



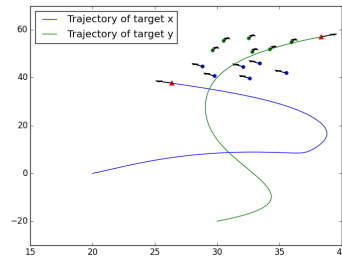
(c)



(d)



(e)



(f)

Figure 3: The trajectory of all agents: (a),(c),(e) sensors controlled by (7). (b),(d),(f) controlled by the algorithm proposed in [21].

less than 20, they can exchange information. The initial distances among all followers are less than 20. The feedback gains of (7) are $c_1 = 2, c_2 = 2$.

$$C_\alpha = \begin{bmatrix} 0 & -0.1 \\ 0.1 & 0 \end{bmatrix}, C_\beta = \begin{bmatrix} -0.16 & 0 \\ 0.1 & 0 \end{bmatrix}. \quad (36)$$

In Fig. 3, it compares the proposed flocking algorithm (7) with flocking algorithm proposed in [21]. By applying this strategy, all followers in the same group can form flock and track its target. But in Figs. 3(b), 3(d), 3(f), it can be seen clearly, sensors in different groups do not separate and track their target.

Figure 4 shows all followers in the same group reach velocity consensus with their targets. In Fig. 5, followers do not reach the same velocity with its target by applying the flocking algorithm proposed in [21]. And there is a big velocity mean square error in the group.

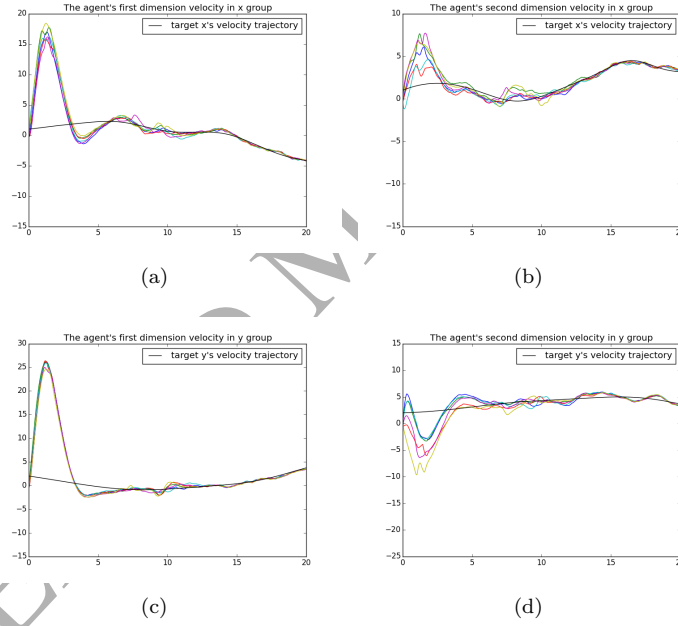


Figure 4: The velocity of every mobile sensors: (a),(c) sensor's velocity in α -group. (b),(d) sensor's velocity in β -group.

In Fig 6, it compares the trajectory of the flock's centre with the trajectory of the target. In this figure, it can be seen that the flock's centre tracks the target well by adopting (7).

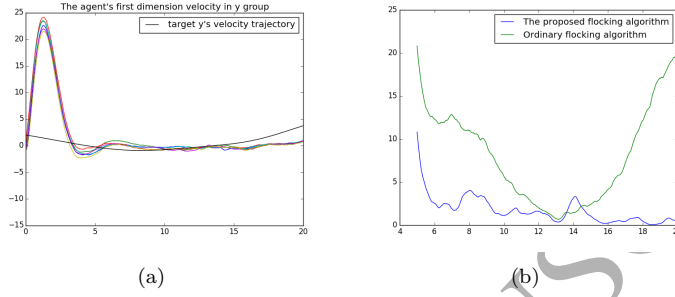


Figure 5: The comparison between two strategies : (a) sensors' velocity by using strategy proposed in [21]. (b) mean square error for distributed target tracking.

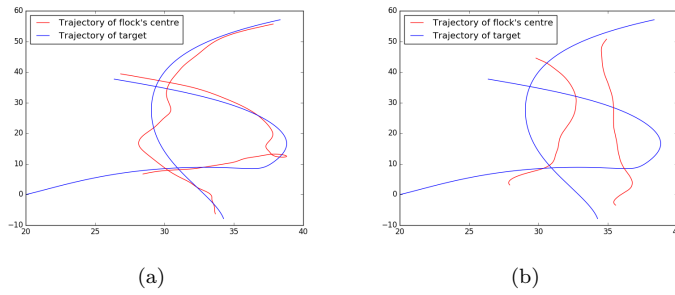


Figure 6: The flocking centre in two situations. (a) Followers are controlled by the (7). (b) Followers are controlled by the flocking algorithm proposed in [21].

6. Conclusion

210 In this paper, we have investigated the mobile sensor networks with two coupled targets. By introducing a new flocking algorithm, all followers in the same group can form a flock and followers in different group are separated. Furthermore, this paper presents the range of feedback gains to make system stable with dynamical estimation errors. Finally, simulations show that, by
215 employing this estimation and control strategy, followers in the same group can be controlled to form a flock and track their target.

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